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A MATHEMATICAL MODEL FOR ECONOMIC AGGREGATE FORECASTING

A THESIS

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ABSTRACT

The objective of this study was to develop a forecasting model to be used by management in predicting the short-run retail gasoline sales of a given firm, its competitors, and the total industry. The purpose of this study was to develop a basis for forecasting the relative competitive positions of retail gasoline firms.

The study was confined to retail gasoline marketing in the State of Georgia, with the short-run prediction period being one quarter. Competition between two or more companies may be measured by several criteria, such as cost, profit, and/or sales. For the purpose of this study, the competitive position of a firm was measured by gallons of gasoline sold by the firm, as compared to the gallons of gasoline sold by its competitors and the total industry.

The companies selected were five of the major firms presently operating in Georgia. For the purpose of this study, they were designated as competitors (1), (4), (6), (8), and (10) ranked in order, with the exception of competitor (6), which was designated as Firm A, the company for relative comparison. A major company was defined as a firm which sold three million gallons of gasoline or more for any given quarter during the four-year period.

The procedure used in achieving the stated objective was divided into two separate analyses. The first developed the total industry-economy relationship, while the second developed the total industry-company

relationship. In the first analysis a relationship between selected economic indicators and total industry gasoline sales was developed. This was obtained from a multiple regression model consisting of three independent variables represented by the economic indicators of electricity consumed, total personal income, and number of telephones in service. The statistical significance of the variables was tested with analysis of variance and the t-test.

The second analysis developed the relationship between total industry gasoline sales and the sales of the five selected companies within the industry. This was achieved by linearly correlating each of the company's sales with the total industry sales.

The relationship of the variables associated with each equation was measured by the simple correlation coefficient. Acceptance of the various equations, supported by the t-test, was based on an average prediction error of five per cent or less, with three per cent or less being preferred. Both of these percentage errors are being used as acceptance criteria by the officials of Firm A, the company being observed. The economic conditions under which the model was developed were assumed to remain relatively stable during the future quarterly periods.

The results of this study were as follows:

(1) Total Industry-Economy Relationship:

(a) Telephones in service and total personal income were found not to be significant. Electricity consumed was found to be significant at the 0.0005 level.

(b) The resulting linear equation, which correlated total industry gasoline sales with electricity consumed, had a correlation coefficient of plus 0.89 and an average prediction error of 2.42 per cent.

(c) Electricity consumed was found to be a coincident indicator for the selected five-year period, necessitating the development of a technique for estimating it one quarter in advance. This was accomplished with base series and demand ratios resulting in an average prediction error of 2.96 per cent.

(2) Total Industry-Company Relationship:

(a) Competitor (10) was the only firm whose sales did not satisfactorily correlate with total industry sales.

(b) The correlation coefficients of the equations representing Firm A, competitors (1), (4), (8), and (10) were plus 0.95, 0.82, 0.96, 0.91, and 0.58 respectively. The average per cent prediction errors for the same equations were 1.58, 2.26, 1.72, 2.37, and 8.75 respectively.

The equations resulting from both analyses, excluding the model representing competitor (10), may be used to predict the corresponding total industry sales and various company sales one quarter in advance. Electricity consumed may be estimated one quarter in advance by using demand ratios and base series.

Recommendations for further study include the following:

(1) Determining the sales response of a selected company to its promotional compaigns and expansionary efforts.

(2) Investigating competitor (10)'s current and past policies and profits for an explanation of the comparatively low correlation coefficient of plus 0.58.

(3) Analyzing gasoline consumption on a long-range basis. Parameters which may possibly affect a long-range study include energy substitutes for gasoline or electricity, transportation substitutes for automobiles, or market saturation due to a limited degree of consumption.

Although the model resulting from this study may be used to predict the competitive position of Firm A, there is no assurance that it would predict with the same accuracy in another industry or in the same industry in a different state or area, or for a different time period.

CHAPTER I

INTRODUCTION

The objective of this study is to develop a forecasting model to be used by management in predicting the short-run retail gasoline sales of a given firm, its competitors, and the total industry. The purpose of this study is to develop a basis for forecasting the relative competitive positions of retail gasoline firms.

The study is confined to retail gasoline marketing in the State of Georgia, with the short-run prediction period being one quarter. Competition between two or more companies may be measured by several criteria, such as cost, profit, and/or sales. For the purpose of this study, the competitive position of a firm is measured by gallons of gasoline sold by the firm, as compared to the gallons of gasoline sold by its competitors and the total industry.

The 1961 sales, as a per cent of the total industry gasoline sales in Georgia, are listed in Table 1 for the ten major competitors, with Firm A being the company for relative comparison [2]. For the purpose of this study, all companies will be designated as competitors (1) through (10), with the exception of competitor (6) which will be designated as Firm A. A major company is defined as a firm that sells three million gallons of gasoline or more for any given quarter during the four-year period.

Table 1. 1961 Sales - Per Cent of Total Industry Sales

Company	Per Cent
1	15.30
2	14.70
3	9.00
4	8.85
5	7.20
6 (Firm A)	7.20
7	6.90
8	4.75
9	2.25
10	1.85

Mathematical forecasting models of several types are continually being applied to industrial situations, and as each method or model illustrates its own unique effectiveness, the mathematical approaches to forecasting advance one more step in supplementing the customary methods of intuition and guessing. If the businessman can observe and measure the past and present phenomena of his firm more closely, using experienced estimates where necessary, he has a more effective means of manipulating and controlling them; and if he can understand the intricate variables which affect his particular firm, he can predict future events of the firm. If he can control the variables, he can control future events. When the entrepreneur knows approximately what to expect, he can plan for these expectations. While there are probably a considerable number of businessmen today who practice this concept, there are probably many others who are not familiar with it [1].

Because of the large number of firms of the gasoline industry, the relative homogenity of their product, and the high degree of knowledge about the market, selling gasoline is a highly competitive business and is likely to be even more so in the future. Since a gallon of gasoline is a low priced item, the key to maximum profit is a high volume of sales. Most of the major oil companies are presently marketing their products in Georgia [3]. Because of the highly competitive environment associated with such a business, management must continually know its past, present, and future position relative to its competitors and to the total industry. Not only is a quantitative measurement of long-run competitive position desired, but also the short-run position.

During recent years there has been a growing public interest in short-term business fluctuations; not only for the overall economy, but also for various industries and companies within the economy. The problems associated with forecasting short-term fluctuations can be numerous and difficult, especially when more than one variable is considered. In annually forecasting one variable as a function of time, the monthly and quarterly variations are smoothed, thereby decreasing the entanglement that is usually encountered by short-term changes. When predicting the short-run position of more than one variable, such as the firm's competitors and the total industry sales, the complexity of determining a reliable forecasting technique increases considerably. Moore [7] states in his recent book, <u>Business Cycle Indicators</u>:

> Many students of current business developments are concerned, not with the cyclical fluctuations of the economy as a whole, but with the prospects of a particular industry, trade, or enterprise. On the numberless detailed tasks of this sort we have

> shed little light, except insofar as the cyclical revivals in the given industry, trade, or enterprise are influenced directly or indirectly by business cycle revivals. Of course this exception constitutes the rule. Nevertheless, cyclical behavior is so variable from industry to industry that an investigator concerned principally with the fortunes of some industry or firm probably will want to find out what other time series, related to his own, have commonly or invariably turned up at earlier dates.

Assumptions

The sales of Firm A (See Table 1) and its competitors are assumed to be dependent on the three following factors: (1) the overall economic growth of the state, (2) the total industry sales within the state, and (3) the promotional and expansionary efforts of the companies within the total industry. This study will not attempt to measure the influence of the third factor on the various firms within the industry, but will assume that the promotional and expansionary actions of any particular firm will be counterbalanced by similar actions of its competitors.

The economic conditions under which the subsequent correlations are developed are assumed to remain relatively stable during the future quarterly periods.

Approach to the Problem

The procedure used in achieving the stated objective was divided into two separate analyses; the first developed the total industry-economy relationship, while the second developed the total industry-company relationship. The procedure followed in each analysis is listed below:

(1) Total Industry-Economy Relationship:

(a) Selection of the independent variables which effectively explain the total industry gasoline sales.

(b) Construction of a multiple regression model of the type

$$Y_{Ti} = a + bX_1 + cX_2 + \cdots + nX_i + \varepsilon_i$$

(c) Determination of the estimates of the constants a, b, c, ..., n, and the significance of their related variables.

(d) Testing the validity of the model using the average error, range of error, presence of regression, and multiple correlation coefficient.

(2) Total Industry-Company Relationship:

(a) Determination of the relationships between the total industry sales and individual company sales of a sufficient number of firms to illustrate the competitive position of Firm A.

(b) Testing the validity of these relationships using the average error, range of error, and simple correlation coefficient.

(c) Testing the validity of relationships (1) and (2) combined into a complete model using the total standard error.

Scope and Limitations

The concept behind a proposal such as that treated here possibly could be applied to any firm; however, the model resulting from this study should be confined solely to the gasoline industry and its companies in the State of Georgia. There is no assurance that it would predict with the same accuracy in another industry or in the same industry in a different state or area, or for a different time period.

The subsequent chapter classifies and discusses the present-day methods of forecasting in order to provide a clearer understanding of their usefulness, applicability, and limitations.

CHAPTER II

RESEARCH OF FORECASTING METHODS

Forecasting may be defined as the analysis of statistical data and other economic, political, and market information for the purpose of reducing the risks involved in making business decisions and long-range plans. Regardless of the behavior of the variable being predicted over a selected period of time, it is important to recognize that any forecasting technique simply provides a means by which the past can be projected into the future. A forecast is apt to be in error to some extent no matter how sophisticated the techniques may be [6].

The literature search indicated that the existing techniques are highly diversified, being strongly influenced by the basic assumptions, the type of data available, and the skill of the individual preparing and presenting the forecast. The majority of the methods were concerned with long-range forecasting on an annual basis rather than the short-term monthly or quarterly periods.

For the purpose of this study, forecasting methods will be classified into the following two groups with related subgroups:

A. Subjective Methods

B. Objective Methods:

1. Trend and Cycle
2. Mathematical Models
3. Simulation

4. Economic Indicators
5. Survey
6. Statistical
7. Combinational

A. Subjective Methods

Since definite statements of what will actually happen is impossible, the subjective methods of forecasting -- intuition, guessing, hunches, judgment, collective-openion, experienced estimates, qualitative knowledge, etc. -- will never be completely eliminated from forecasting practices.

A survey in 1956 by the American Management Association [1] of 297 companies representing a cross-section of all industry in the United States, indicated that the majority of firms still base their forecasts on judgment. Another survey in 1958 by Dun's Review and Modern Industry [8] of 77 companies representing a similar cross-section is summarized by the following statement:

> Highlighting the whys and hows of company forecasting practices, this new survey reveals that some top management are still putting as much faith in hunch and intuition as in statistics and economic indicators.

Based on these two surveys, it appears that the subjective approaches to forecasting are still used by a large number of industrial firms. This is not to imply these are the best methods. All of the subjective methods have the same basic limitation, namely, they are not quantitative. Most, if not all, forecasters realize, however, that their judgment is no better than the information on which it is based, and they actively seek assistance from all the available sources [9].

B. Objective Methods

The objective methods of forecasting usually provide a more scientific approach, but experience has proved there is risk involved in blindly accepting a quantitative expression.

Trend and Cycle

This method of forecasting is related to the theory of trends and cycles described by Dakin and Dewey [10] and Gordon [11].

Dakin and Dewey relate the biological growth of nature and organisms to the growth of human institutions and business organizations. They believe that a growth trend amounts to a pattern, and the pattern is similar for almost all organisms, whether a group of cells in a plant or the cells that make up business firms. If trends have continued for decades, or if the oscillations of cycles around the trend have repeated themselves so many times and so regularly that the rhthym cannot reasonably be the result of chance, there is a strong possibility that these behaviors will continue. The business conditions were believed to be influenced by four major rhythmic cycles, namely, the 54, 18, 9, and 3-1/2-year rhythms. By projecting these cycles into the future, the fluctuations of the economy were predicted.

Gordon [11] states, however, that forecasters today claim business cycles are not strictly periodic; there is no conclusive evidence that a long cycle of 50 or 60 years exists; and short-run and long-run cycles can be combined in many varied ways. He also states that there has been no complete major cycle in the United States since 1933.

Mathematical Models

A common technique for solving technical problems is to construct a model of a machine or activity, which can be used to simulate its operations and thereby to predict its behavior in an actual situation.

Lindsay [12] explains that in mathematical analysis, the word "model" is used to mean a mathematical description of an activity which expresses the relationships among the various elements with sufficient accuracy that it can be used to predict the actual outcome.

There have been several mathematical models developed for the overall economy. Klein [13] constructed a 25-equation model of the United States economy that performed exceptionally well in 1953 and 1954. Clark [14] developed a model containing 80 mathematical relationships from which an electronic computer can extract a forecast of the United States economy for 18 months in advance. The model gives quarterly forecasts of approximately seventy statistical series and a complete new forecast can be produced in two days. Some authors refer to these models as econometric models.

Stillson and Arnoff [15] developed procedures which may be used in constructing a mathematical model to study certain marketing problems and the effects of marketing decisions on the firm. Specifically, an attempt is made to show how the marketing manager may study the problems of product research and evaluation, with special emphasis on how to develop a simple break-even model to analyze the related variables.

Vidale and Wolfe [16] present a model of the interaction of advertising and sales performed on a large number of products. The model is based on three parameters: sales decay constant, saturation level, and response constant, and has proved useful for the analysis of advertising campaigns and the allocation of advertising expenditures.

The previously mentioned models are of many types, and their characteristics depend on the real-life situation they simulate. The obvious advantage of a model is that it, instead of the organization it simulates, can be manipulated in a variety of ways until the best solution is found. The great disadvantage is that no model can completely duplicate reality; therefore, both the construction of the model, and the translation of the results to the actual problem must be done with extreme caution.

Simulation

Lindsay [17] described simulation as a systematic trial-and-error procedure for solving complex problems, offering the possibility of comparing many alternative courses of action and measuring each course on its relative merits. He explains how an exact mathematical solution will probably be impossible because of the complexity of the particular problem, and because many of the individual factors will not vary according to simple mathematical formulas. An example is presented illustrating the risks involved in making bids for a construction operation, i.e., bidding too high or too low relative to the estimated time to complete the operation; however, he states that it is equally possible to use simulation for evaluating results in terms of costs, efficiency, share of the market, sales, or any other criterion or set of criteria.

Forrester [18] developed a digital computer simulation for an industrial firm to illustrate how a sudden increase or decrease in retail sales affected the order rates, factory output, warehouse inventory, and unfilled orders.

Even though the advent of the high-speed computers has extended tremendously the capabilities of simulation processes, the costs of data collection, computer programming, and computer operation are substantial; and the potential saving must be sufficiently attractive to justify the expense. Also, construction of the model takes time; therefore, simulation cannot be applied to crash programs. The great advantage of simulation is that it is not restricted by the requirement that the model be solved mathematically. It is this requirement that places a limit on the usefulness of many of the strictly mathematical solutions to management problems.

Economic Indicators

Business forecasters have long sought for an index or indexes which would change consistently before some other index which they wished to forecast; however, in spite of the strides that have been taken, there is still no infallible indicator of business activity.

The Harvard Business School in the 1920's constructed several series which were supposed to indicate what was ahead in business and finance. In 1938 Roos examined 248 monthly indexes to determine which, if any, had lead characteristics. In 1954 he observed that the series found to have consistent lead characteristics in 1938, still had the same characteristics [19].

In 1937, Mitchell and Burns [20] studied 487 statistical series in a monthly and quarterly form. From these, twenty-one indicators were determined to be the most reliable.

In 1950, Moore [21] examined 801 monthly and quarterly indicators for the United States to determine which were consistently early and which late in a business cycle. From this study resulted a published list of twenty-one indicators classified into three groups--leading, roughly coincident, and lagging--according to their tendency to reach cyclical turns ahead of, at about the same time as, or later than business cycle peaks and troughs. A new list, selected on the basis of the same criteria, includes twenty-six indicators, of which twelve are classified as leading, nine as roughly coincident, and five as lagging. It was interesting to note that of the twenty-one indicators selected by Mitchell and Burns in 1957, only three remain on Moore's new list: business failure liablilities, industrial production index, and average workweek in manufacturing [22].

In order to obtain a better understanding and feel for the ups and downs of business indicators, the National Bureau of Economic Research developed the diffusion index. In essence, a diffusion index simply measures the percentage of time series in a given sample expanding each month. For example, one that measures 21 indicators registers 66.7 per cent for any month in which 14 of them rise. The curve is at zero if all are declining, at 50 per cent when half are rising and half are declining, and at 100 per cent when all are rising [23].

Even though diffusion indexes provide useful summaries of current economic events, they have two main disadvantages:

(1) Although they indicate every general economic turning point, their sensitivity leads them to indicate also some that never occur.

(2) More important, they interpret economic events too mechanically.

C. Ashley Wright [24] of the Standard Oil Company of New Jersey, developed "Wright's Indicator" which is designed to determine turns in the business cycle. He discovered that the upturns and downturns of forty deliberately selected indicators tend to cluster in a normal distribution, and that the highs and lows of the curve point out the turning points in the general economy.

Economic indicators have two major limitations: (1) They cannot predict when a business turning point will occur, and they usually recognize a new upswing or downswing only after it has started, and (2) they can give no idea of how intense a revival or recession will be, or its duration until it is well under way.

Survey

Of all the objective methods reviewed in this study, the survey is probably the least quantitative. Some of the companies and agencies that conduct surveys include the following: (1) Fortune magazine, (2) Dun and Bradstreet, (3) The United States Department of Commerce and the Securities Exchange Commission, (4) McGraw-Hill Book Company, (5) Survey Research Center at the University of Michigan, (6) Surveys of Consumer Finance, (7) Federal Reserve System, and (8) Bureau of Economic and Business Research of the University of Illinois.

The American Management Association [1] conducted a survey to determine what were the most common sources of economic information used by companies in surveying consumer expectations. The most popular sources are ranked in the following order: (1) Survey of Current Business, (2) Business Week, (3) Federal Reserve System, (4) Wall Street Journal, (5) the Econometric Institute, (6) National Industrial Conference Board, (7) Kiplinger Washington Agency, and (8) Sales Management.

Dun and Bradstreet [25] conducted 16 surveys of business expectations between 1947 and 1951. The response to these surveys were similar and sizably wrong.

The Surveys of Consumer Finance [26] involved consumer anticipations for refrigerators, furniture, radios, and television sets. The overall predictive efficiency of these items was generally poor.

Associates of the Wharton School of Finance claim to be nearing completion of a quarterly forecasting model using a high speed computer that will use anticipations data from surveys of consumer and business spending plans [14].

Ferber [27] made a study on the reliability of responses obtained in surveys from which were developed three propositions:

(1) The degree to which the attitudes and expectations of one member correlates with those of other family members is low.

(2) A particular family member generally does not have complete knowledge of the purchasing habits of other family members.

(3) In a large number of families, information obtained regarding family status and characteristics will differ depending on which family member is interviewed.

One obvious and basic limitation of the survey method is that it requires considerable judgment in its application. Other limitations are the lack of clarity in the consumer's mind and in his reporting of his expectations when plans are made. There is also a limitation in regard to the time lag between the start of interviewing and availability of the forecast.

Statistical

The statistical methods of forecasting utilize regression and correlation analysis to predict the future position or quantity of a dependent variable. Regression equations usually contain one or more independent variables, and one dependent variable. A correlation between two or more variables may either be linear or nonlinear, depending on its applicability and effectiveness in predicting the dependent variable.

Magee [6] describes a method in which a leading index is found to which company sales are highly correlated. He states that if the index does not lead the dependent variable, the company is left with the problem of predicting the index itself; even though this adds complexity to the problem, this does not mean the index becomes useless. Forecasters have accurately and reliably predicted coincident indicators, then in turn used this prediction as the independent variable to forecast their unknown dependent variable. Klein [11] [28] points out some indicators are known in advance with a relatively high degree of accuracy; however, there is no guarantee that a time lag will continue, just as there is no guarantee that the relationship between the coincident indicator and dependent variable will continue.

Brown [4] and Winters [5] both use the statistical method of exponentially weighted moving averages to forecast on a weekly or monthly basis. Brown uses one smoothing constant whereas Winters uses three smoothing constants considering seasonal and trend effects simultaneously. Davis [29] applied Brown's method to the demand for hospital supply items with the results being a practical forecasting model that would effectively predict the monthly and weekly demand for hospital gloves.

Cotter [30] uses a technique whereby he compares the trends from the regression equations of his dependent and independent variable; then measures the per cent deviation from each of the variables trends and develops a correlation between the two deviations. He uses a coincident indicator as his independent variable and makes the assumption that the indicator can be accurately predicted.

Sonnendecker [31] develops a statistical model to forecast the blood requirements of a hospital by mathematically combining frequency distributions that represent the number of blood units drawn per unit of time. The author assumes normality, thereby enabling the distributions to have the property of additivity of the means and variances.

Eilon [32] presents the use of a control sales chart with a regression line. When the actual sales figures are ascertained, their deviations from the mean trend can be checked by the control chart. If the points are in control, the fluctuations can be attributed to chance causes and the method for forecasting may remain unchanged. If the point falls outside the control lines, the reason for this deviation from the trend must be determined and the general trend re-evaluated.

Combinational

After reviewing the existing techniques of forecasting, one may readily state that the various methods should not be mutually exclusive, even though a large percentage of the companies use them in this fashion. There are much greater benefits to be obtained when a company can base its decisions from the results of several techniques than from a single method. Both Magee [6] and Eilon [32] suggest a combination of methods a business firm may easily apply.

Weinberg [33] illustrates an example of combining several methods of forecasting in developing a long-run multiple factor break-even analysis for a bottling company. He first obtains an indicator that effectively predicts the total industry sales. He then uses linear and curvilinear regression techniques to develop relationships between the total industry sales and company profits before and after taxes. Utilizing these correlations, he measures the effect of the past and present advertising programs on the company profits, derives growth and decay functions using past trends, then suggests a series of surveys to estimate the promotional expenditures of the company's competitors. This, he explains, would give him an estimate of the future promotional plans of his competitors. He does not explain, however, exactly how to conduct such a survey in obtaining this information.

It would be rather difficult to find a problem that could be solved by using all eight of the previously discussed forecasting methods. In order to achieve the stated objective of this study, the statistical techniques of regression and correlation will be used to establish an effective relationship between certain economic indicators and the sales of selected firms. The subsequent chapter analyzes the relationship between total industry gasoline sales and specific economic indicators.

CHAPTER III

TOTAL INDUSTRY-ECONOMY RELATIONSHIP

The objective of this analysis is to develop a technique for predicting total industry gasoline sales one quarter in advance. The analysis will be limited to two forecasting methods, namely, economic indicators and multiple regression. The economic indicators will be the independent variables, while total industry gasoline sales will be the dependent variable. In attempting to quantify the relationship between the selected economic indicators and the total industry sales, the variables are classified as follows:

Dependent Variable

Y_{Tij} = gasoline sales in Georgia in millions of gallons during the i^{th} quarter and j^{th} time period.

Independent Variables

X_{1ij} = electricity consumed in millions of kilowatt hours during the i^{th} quarter and j^{th} time period.

X_{2ij} = total personal income in millions of dollars during the i^{th} quarter and the j^{th} time period.

X_{3ij} = telephones in service in thousands of telephones during the i^{th} quarter and the j^{th} time period.

X_{4ij} = total number of vehicles registered during the i^{th} quarter and j^{th} time period.

Both the first and third variables, electricity consumed and telephones in service, are growth indicators. Since both are a measure of the economic growth and population influx into the state, as is total gasoline sales, it was felt that either or both may produce a high enough correlation to be used in predicting total industry gasoline sales.

The data related to total industry gasoline sales were obtained from representatives of the petroleum industry and the Petroleum Council of Georgia. The data concerned with electricity consumed were obtained from Georgia Business, which is published by the Bureau of Business Research, Athens, Georgia. Information related to telephones in service was obtained from the Southern Bell Telephone and Telegraph Company of Atlanta.

Total personal income provides a measure of the short-run change in the purchasing power of the consumer. The various components of personal income, as determined by the Federal Reserve Bank of Atlanta, are estimated separately from wage and salary payments, agricultural income, proprietor's income, property income, and transfer payments. Since seasonal movements dominate the short-run fluctuations of many economic indicators, the series must be seasonally adjusted. In the case of personal income, agricultural income causes the majority of the short-run variations. For this reason alone, it was felt that seasonally adjusted data would be more representative of the purchasing power of the consumer relative to gasoline. The personal income data were obtained from the Federal Reserve Bank of Atlanta.

The fourth variable, total number of vehicles registered during the i^{th} quarter, represents the major consumer of gasoline. "Vehicles" includes all automobiles and trucks operating on a commercial, personal, and/or public basis. This would include vehicles operated by the State and Federal Government, military, and county officials. The information concerned with this variable was obtained from the Federal Reserve Bank of Atlanta and the License Tag Information of the State Revenue Department. After plotting this datum on a quarterly basis, it was observed that over 50 per cent of the observations were concentrated in the first quarter, and over 80 per cent in the first two quarters for any given year. After further investigation, this relationship was explained by the fact that state regulations, supplemented by a penalty fee, required all vehicles to be registered prior to March 31st during any year. Due to this reason, the datum was considered as being unusable for the purpose of this analysis and eliminated from the model.

The data pertaining to the remaining variables are listed in Table 2. In order to determine the individual influence of each independent variable, time series diagrams of Y_{Tij}, X_{1ij}, X_{2ij}, and X_{3ij} were plotted in Figures 1, 2, 3, and 4 respectively. After comparing each diagram of the independent variables with the diagram of the dependent variable, the time period of $j = 0$ for all four variables appeared to be the most feasible. Because of this mutual relationship, the zero will be omitted in subsequent equations.

The proposed multiple regression model in Equation (1) assumes that total industry gasoline sales are a linear function of each independent variable.

Table 2. Data for Regression Analysis

Quarter	Y_{Ti}	X_{1i}	X_{2i}	X_{3i}
1957 - 1	279	1,221	16,250	584
2	302	1,306	16,326	590
3	301	1,446	16,285	597
4	288	1,236	16,231	603
1958 - 1	281	1,281	16,672	606
2	305	1,289	16,715	608
3	310	1,494	17.206	615
4	305	1,386	17,471	623
1959 - 1	295	1,401	17,777	632
2	327	1,468	18,134	638
3	337	1,712	18,354	650
4	304	1,496	18,584	660
1960 - 1	305	1,499	18,770	669
2	335	1,569	19,079	674
3	333	1,856	19,126	683
4	324	1,616	19,159	695
1961 - 1	309	1,565	19,276	696
2	337	1,643	19,331	699
3	353	1,880	19,667	706
4	330	1,705	20,466	713

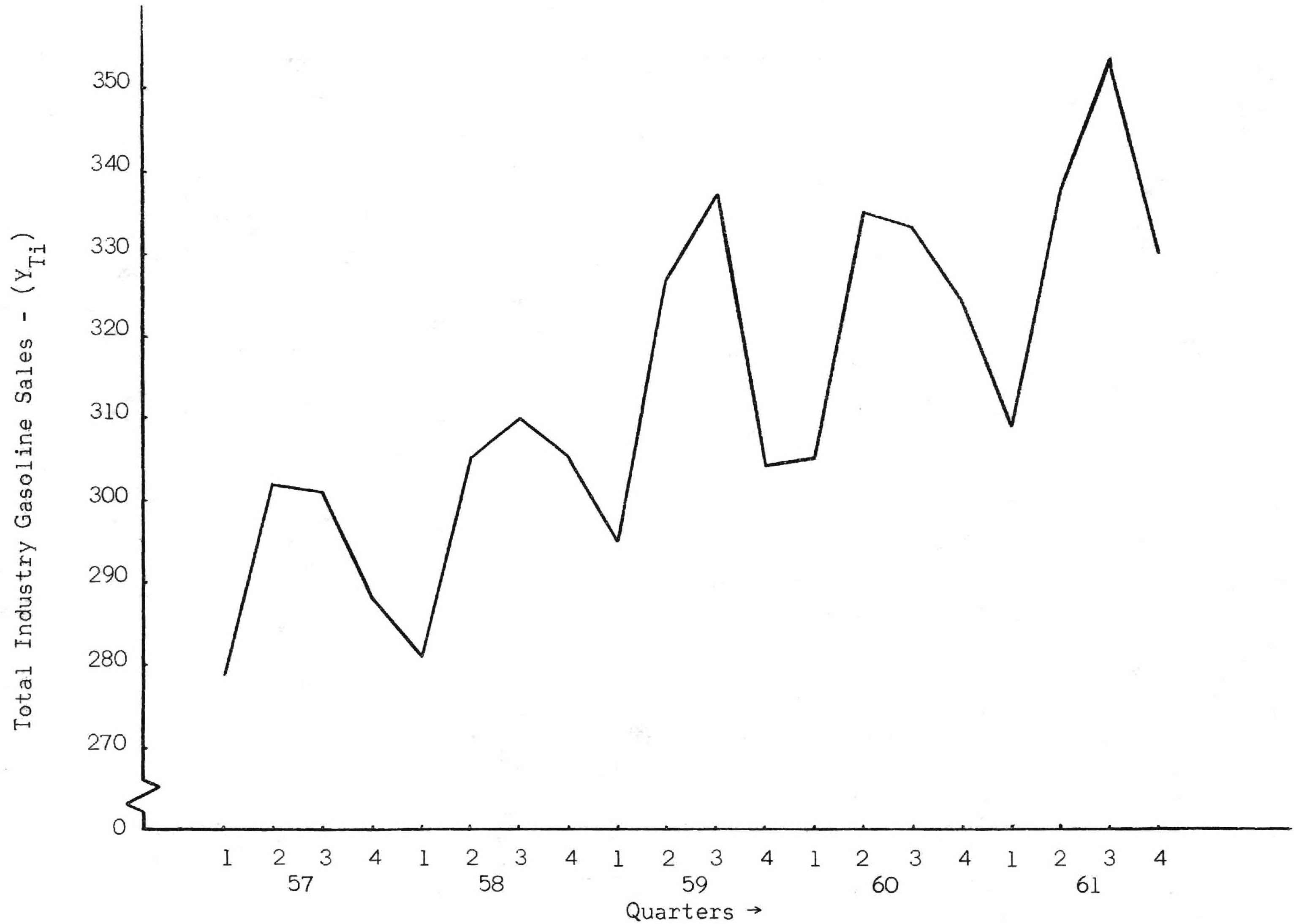

Figure 1. Determination of Time Periods - Y_{Ti}

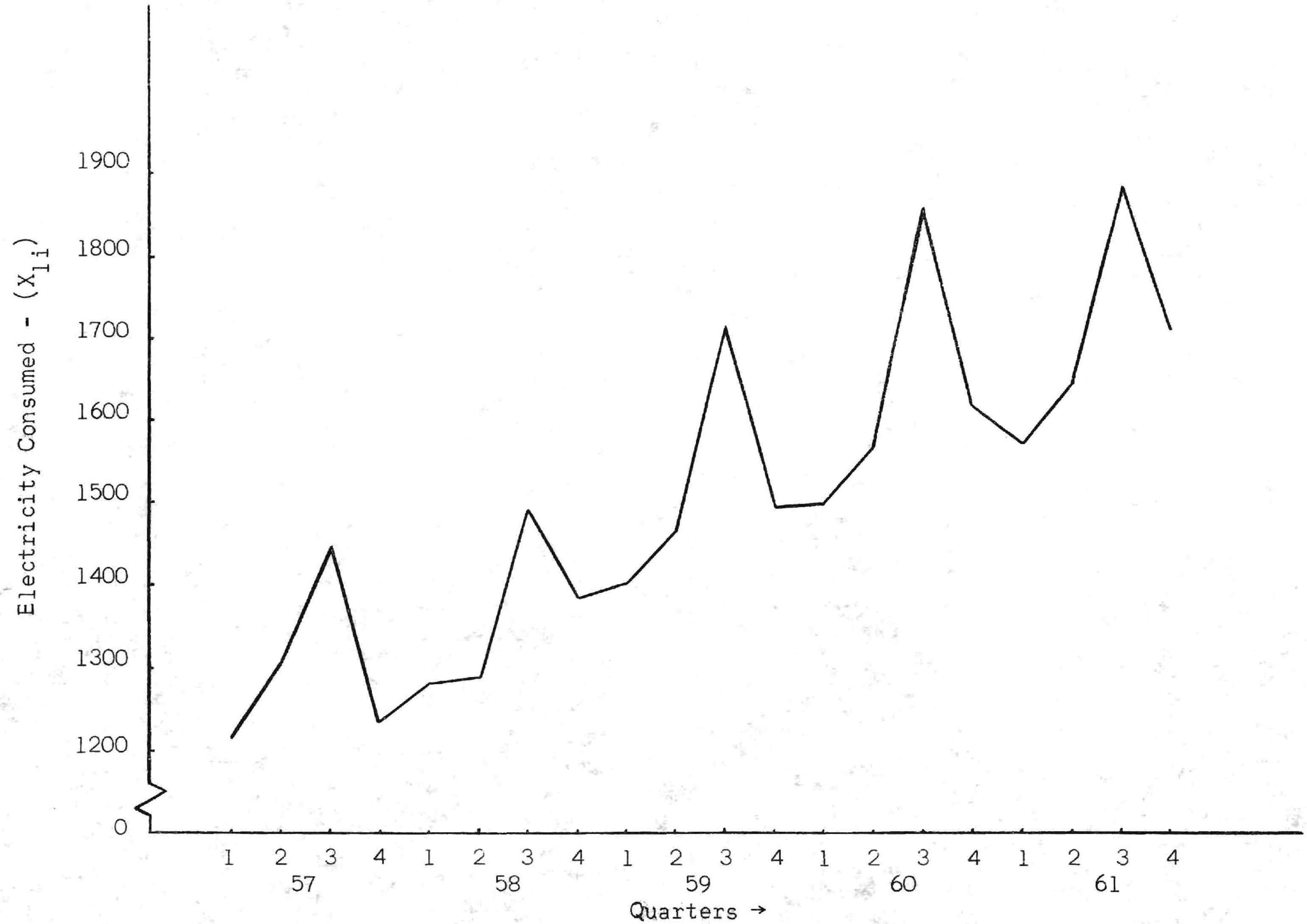

Figure 2. Determination of Time Periods - X_{1i}

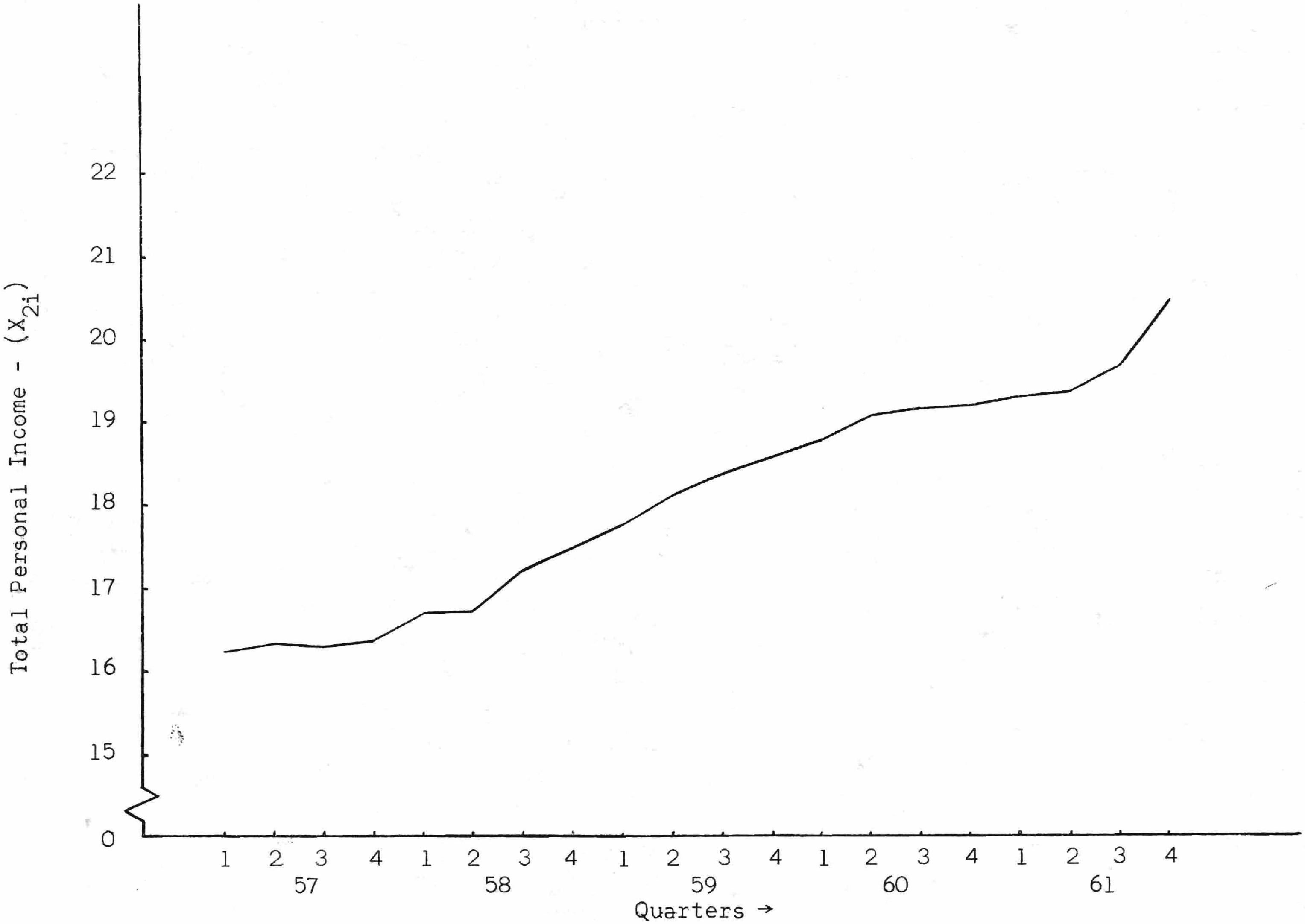

Figure 3. Determination of Time Periods - X_{2i}

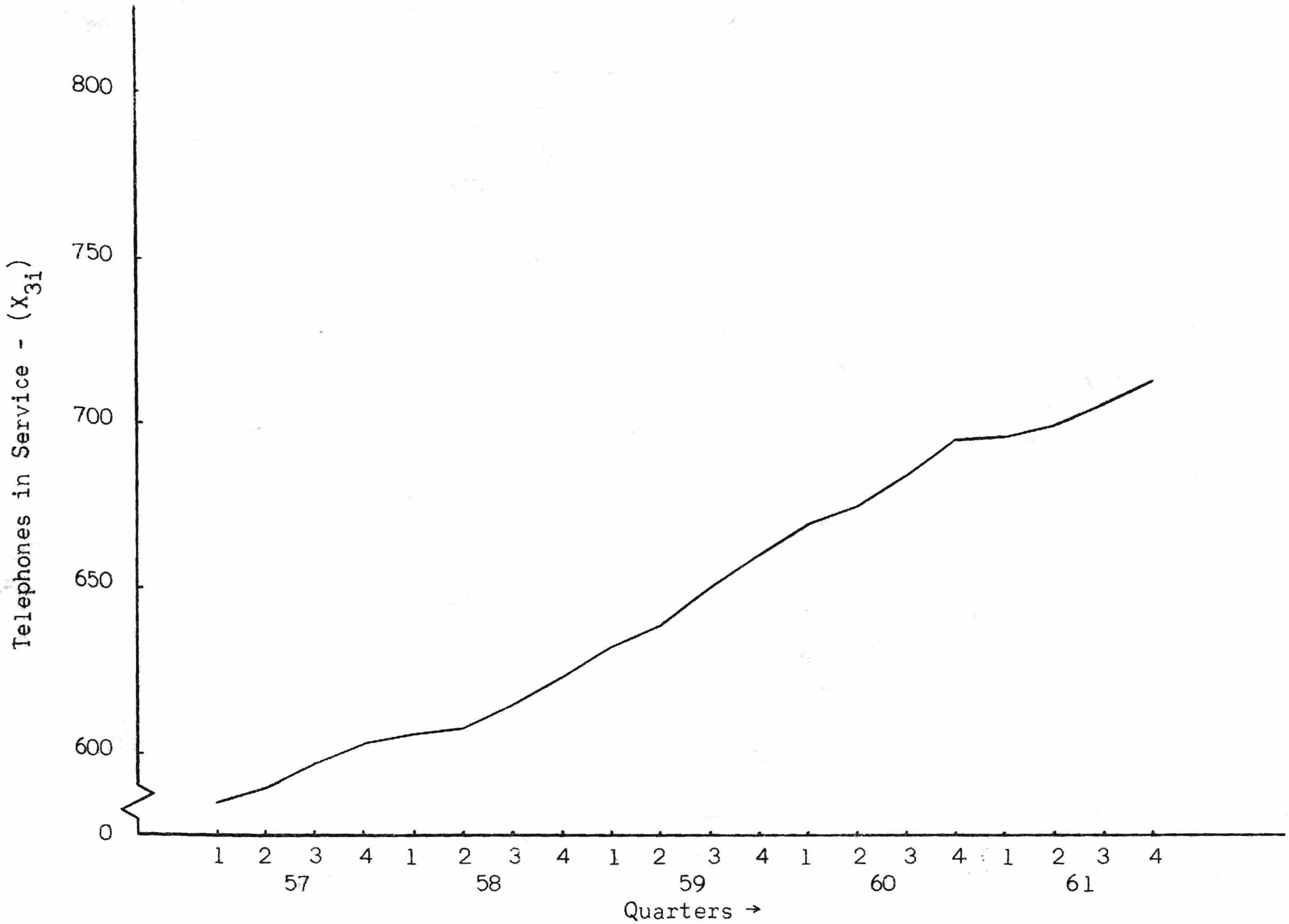

Figure 4. Determination of Time Periods - X_{3i}

$$Y_{Ti} = AX_0 + BX_{1i} + CX_{2i} + DX_{3i} + \varepsilon_i \quad (1)$$

where A, B, C, and D are the true regression coefficients for each related independent variable, and ε_i is the random error independent of the X's. In most cases this last element is not truly random, because it includes some factors that could be accounted for if one had the time and patience to investigate them. However, it is usually not inappropriate to consider it to be a random variable following some assumed probability distribution.

The Burroughs 220 computer routine MRS-017 was used in determining the correlation and regression statistics listed in Table 3. The least squares multiple regression equation for the model represented by Equation (1) is given as follows:

$$\hat{Y}_{Ti} = aX_0 + bX_{1i} + cX_{2i} + dX_{3i} \quad (2)$$

where $\hat{Y}_{Ti} - Y_{Ti} = e_i$ = residual error for the i^{th} quarter.

The results from the multiple regression analysis produced the following equation:

$$Y_{Ti} = 171.94 + .0895X_1 + .00593X_2 - .154X_3 \quad (3)$$

The multiple correlation coefficient for Equation (3) is calculated in Appendix I, and is $R = .90$.

After observing the partial correlation coefficients of Table 3, there is little doubt that X_1 is the more effective predictor of Y_{Ti}. Subsequent statistical tests will support this statement.

Table 3. Multiple Regression Analysis Results

		Y_{Ti}	X_{1i}	X_{2i}	X_{3i}
Averages		313	1,504	18,049	647
Standard Deviation		20.4	193	1,318	42.8
Simple Correlation Coefficients	r_{YY}	1.000	0.892	0.776	0.764
	r_{X_1Y}	0.892	1.000	0.846	0.845
	r_{X_2Y}	0.776	0.846	1.000	0.984
	r_{X_3Y}	0.764	0.842	0.984	1.000
Regression Coefficients	b_k		0.0895	0.00578	- 0.154
Partial Correlation Coefficients	r_{Y_k}		0.707	0.146	- 0.124
Standard Error of Regression Coefficients	S_{b_k}		0.0224	0.0100	0.3107

Snedecor [34] uses analysis of variance in testing the regression of the complete equation and its related independent variables. The first hypothesis to be tested is that no regression is present in the complete Equation (3), that is: $H_0 : B = C = D = 0$. The results of this analysis are listed in Table 4.

Table 4. Analysis of Variance Summary

Source of Variation	SS	df	MS	F-Ratio
Regression on X_1, X_2, and X_3	6,668	3	2,223	28.10
Regression on X_1 and X_2	6,606	2		
Regression on X_3 after X_1 and X_2	62	1	62	0.784
Residual Error	1,266	16		
Total	7,934	19		

The F-value of 28.10 in Table 4 is significant at the .001 level and indicates that regression is present in the population from which the sample was drawn. In essence, this is rejecting the null hypothesis. All computations related to Table 4 are in Appendix I.

The second hypothesis to be tested using the results of Table 4 is: $H_0 : D = 0$. This will tell whether X_3 contributes significantly to the prediction of Y_{Ti}.

The F-value of 0.784 is not significant; therefore, the null hypothesis is accepted and we conclude that we have insufficient evidence that X_3 contributes to the prediction of Y_{Ti}. This also supports the low value of the partial correlation coefficient of X_3 in Table 3.

The t-test will be used to test the significance of the remaining variables X_1 and X_2. The hypothesis to be tested will be $b = c = B_k = 0$. If the

$$|t_k| = \frac{(b\ ,\ c) - B_k}{S_{b_k}} \geq t\ \alpha/_{2;N-n}, \quad \text{reject the hypothesis.} \qquad (4)$$

Where $df = N - n = 20 - 2 = 18$

N = number of observations

n = number of independent variables

S_{b_k} = standard error of the k^{th} regression coefficient for $k = 1, 2$.

The test statistic, t, with 18 degrees of freedom at the 90 per cent confidence level is 1.734. The results of the t-test are listed in Table 5 while the calculations are in Appendix I.

Table 5. Results of t-test

Independent Variable	Absolute t-value	Significance
X_1	33.280	Yes, at $\alpha/2=.0005$
X_2	0.013	Not significant

After removing the independent variables which do not contribute significantly to the prediction of Y_{Ti}, the resulting equation is

$$Y_{Ti} = 170.73 + .0946X_1 \tag{3}$$

Figure 5 illustrates a graphical relationship of Equation (5).

The 95 per cent confidence intervals for three values of X_1 in Equation (5) are listed in Table 6 and calculated in Appendix I. It may also be observed from Figure 5 that all of the observations lie within the confidence interval. In the long-run, no more than five per cent of them would be expected to fall outside. The $\bar{X}_i$ value illustrated in the same figure indicates that the closer the predicted values of the independent variable X_1 comes to $\bar{X}_1$, the better the estimate of Y_{Ti} will be.

Table 6. 95 Per Cent Confidence Intervals

X_1	Confidence Intervals at $\alpha/2 = .025$
1300	294 ± 19.0
1600	322 ± 18.6
1900	350 ± 20.3

Table 7 illustrates the relative effectiveness of the complete Equation (3) and the reduced Equation (5). The calculations of the residual error for both equations are listed in Tables 14 and 15 in Appendix I.

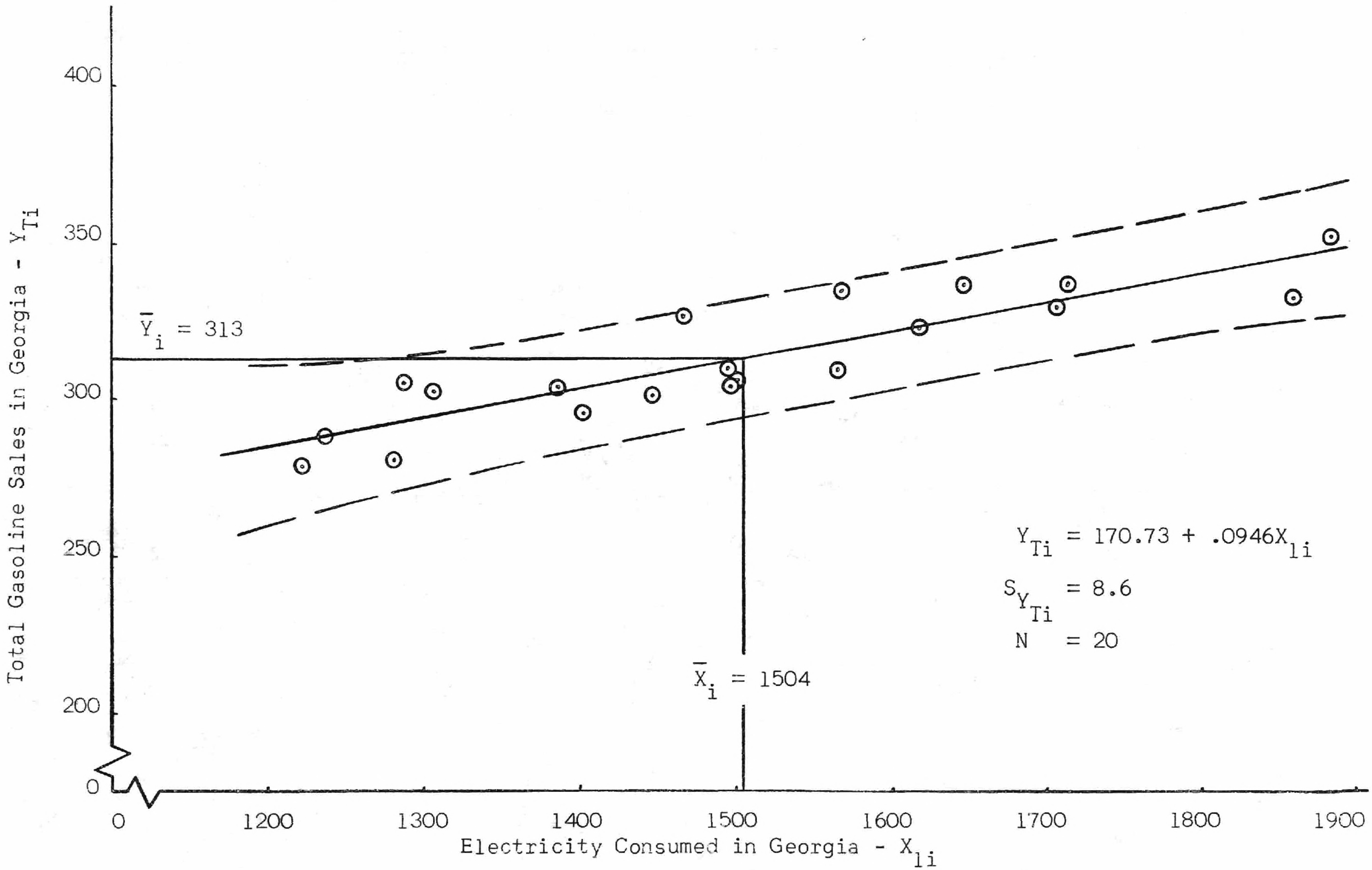

Figure 5. Illustration of Equation (5)

Table 7. Comparison of Equations (3) and (5)

Equation	R	R^2	Average Error	Average Error,%	Range of Error
(3), Complete Model with X_1, X_2, X_3	0.90	0.81	7.85	2.51	27.8
(5), Reduced Model with X_1 only	0.89	0.80	7.59	2.42	30.8

An average prediction error of less than five per cent is usually accepted by the petroleum industry, with equal to or less than three per cent being preferred. Based on either criterion, both Equations (3) or (5) could be used for predicting purposes. However, since practicality is an important criterion to consider, and since the other measures of effectiveness in Table 7 are approximately equal, the reduced model may be used in predicting the dependent variable Y_{Ti}.

Estimate of Electricity Consumed

Since electricity consumed is a coincident indicator, it becomes necessary to estimate it in order to forecast Y_{Ti}. In essence, we are forcing a lag relationship between the dependent and independent variable.

Because of the peak and trough consistancy of electricity consumed, it was felt that estimating with demand ratios might be applicable. This technique makes use of a base series, which must be determined by trial and error. A base series, as defined by Brown [35], compares a single demand or average of demands in previous periods with the demand in a present corresponding period. If the base series is 3, this would mean that the surrounding three quarters in the previous year would be averaged, then compared with the present corresponding quarter. The base series with the least variation from the pattern of demand gives the best results.

The demand ratios for base series of 1, 2, and 3 quarters are plotted in Figure 12 in Appendix I. The base series of 1 appeared to fluctuate the least; however, to quantify the observation, all three series were tested with the results listed in Table 8. The base series of 1 obviously gives us the best estimate of electricity consumed. The demand ratios and residual error are calculated in Table 16 in Appendix I. The equation below may be used to predict electricity consumed one quarter in advance. Although a linear projection such as this is comparatively effective, the author does not recommend extrapolation in excess of three months because of the greater chance of significantly deviating from the trend which has already been analyzed quarterly.

$$\text{Forecast}_{(t+1)} = \text{Demand Ratio}_{(t)} \times \text{Base Series}_{(t+1)} \tag{6}$$

Table 8. Trial and Error Selection of Base Series

Base Series, Quarter	Standard Error of Estimate	Average Error	Average Error,%
B = 1	58.8	46.0	2.96
B = 2	133.8	113.5	7.30
B = 3	176.5	147.5	9.50

Utilizing Y_{Ti} as an independent variable in lieu of a dependent variable, the following chapter develops the relationships between total industry gasoline sales and the selected company sales.

CHAPTER IV

TOTAL INDUSTRY-COMPANY RELATIONSHIP

This analysis is concerned with the development of a technique to predict the competitive position of Firm A relative to the total industry one quarter in advance. The forecasting method of linear regression will be used to establish a correlation between the separate company sales and the total industry sales during any given quarter. The variables are defined as follows:

Dependent Variable

Z_{ci} = selected company gasoline sales in millions of gallons during the i^{th} quarter.

Independent Variable

Y_{Ti} = total industry gasoline sales in millions of gallons during the i^{th} quarter.

Competitors (1), (4), (8), and (10) were considered to be an adequate number of companies to describe Firm A's sales position relative to its competitors and the total industry. The quarterly sales data of the selected competitors are listed in Table 9. In order to ascertain the influence that total industry sales may have on the selected company sales, graphical relationships for company sales versus total industry sales are illustrated by Figures 6, 7, 8, 9, and 10.

Table 9. Data for Bivariate Regression Analysis

Quarter	Y_{Ti}	Company Sales, Thousands of Gallons (Z_{ci})				
		Competitor (1)	Competitor (4)	(Firm A)	Competitor (8)	Competitor (10)
1958 - 1	281	46,748	24,594	19,479	10,526	6,320
2	305	50,738	26,933	20,571	11,565	6,437
3	310	50,051	26,855	21,529	11,794	6,045
4	305	48,559	26,779	21,297	12,247	6,143
1959 - 1	295	46,923	25,697	20,779	13,528	5,263
2	327	52,289	28,300	22,625	15,427	5,617
3	337	51,817	29,824	22,952	16,200	5,762
4	304	46,462	27,211	21,431	13,677	5,179
1960 - 1	305	47,663	27,564	20,875	14,051	5,289
2	335	52,454	30,656	23,095	15,920	6,664
3	333	51,424	31,063	23,190	18,492	6,923
4	324	49,269	29,315	22,789	15,806	6,885
1961 - 1	309	47,891	28,584	21,970	15,006	6,778
2	337	51,436	30,702	24,477	16,371	7,999
3	353	54,283	31,469	24,926	16,365	7,523
4	330	49,477	29,782	24,230	14,984	7,266

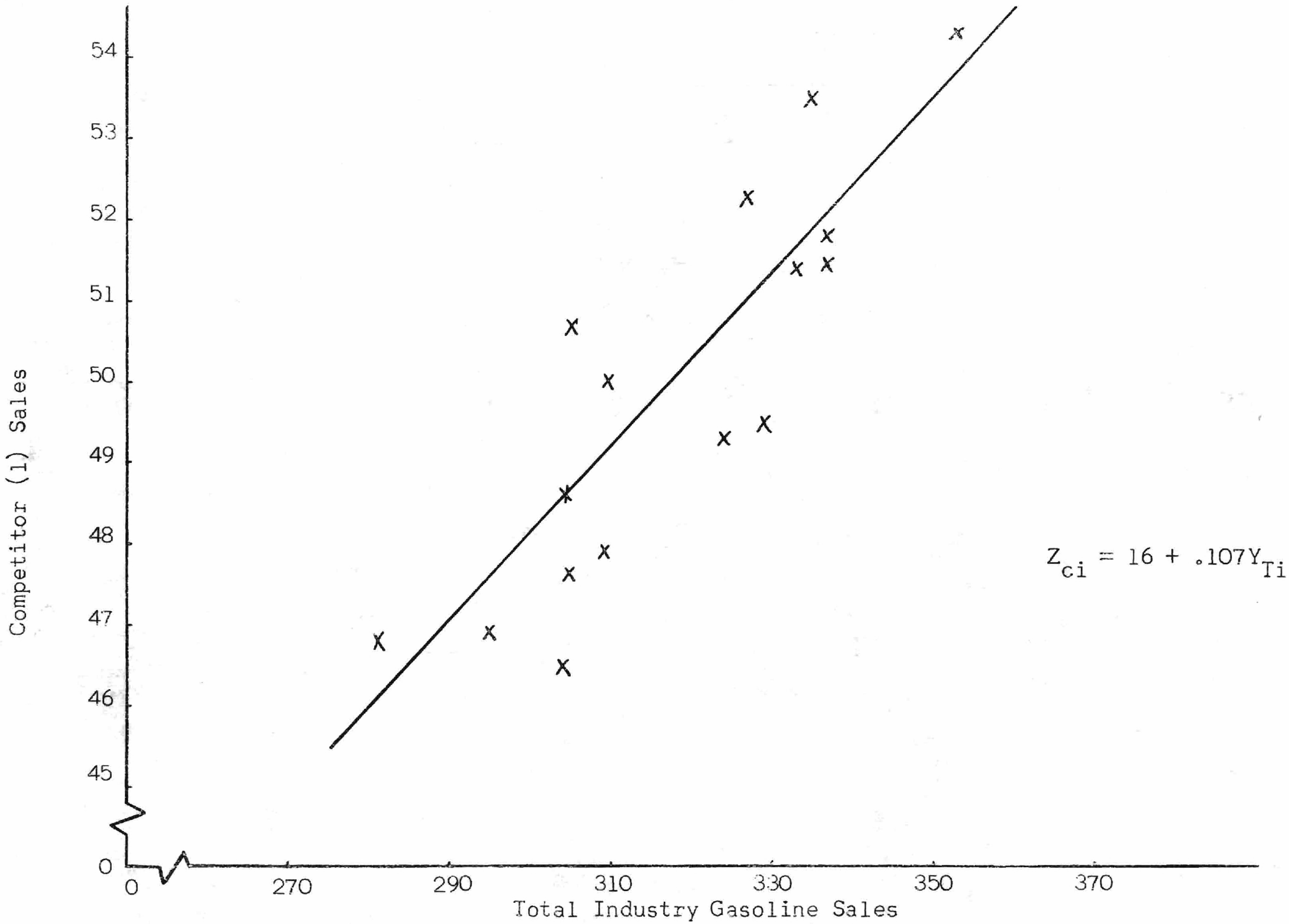

Figure 6. Competitor (1) Versus Total Industry

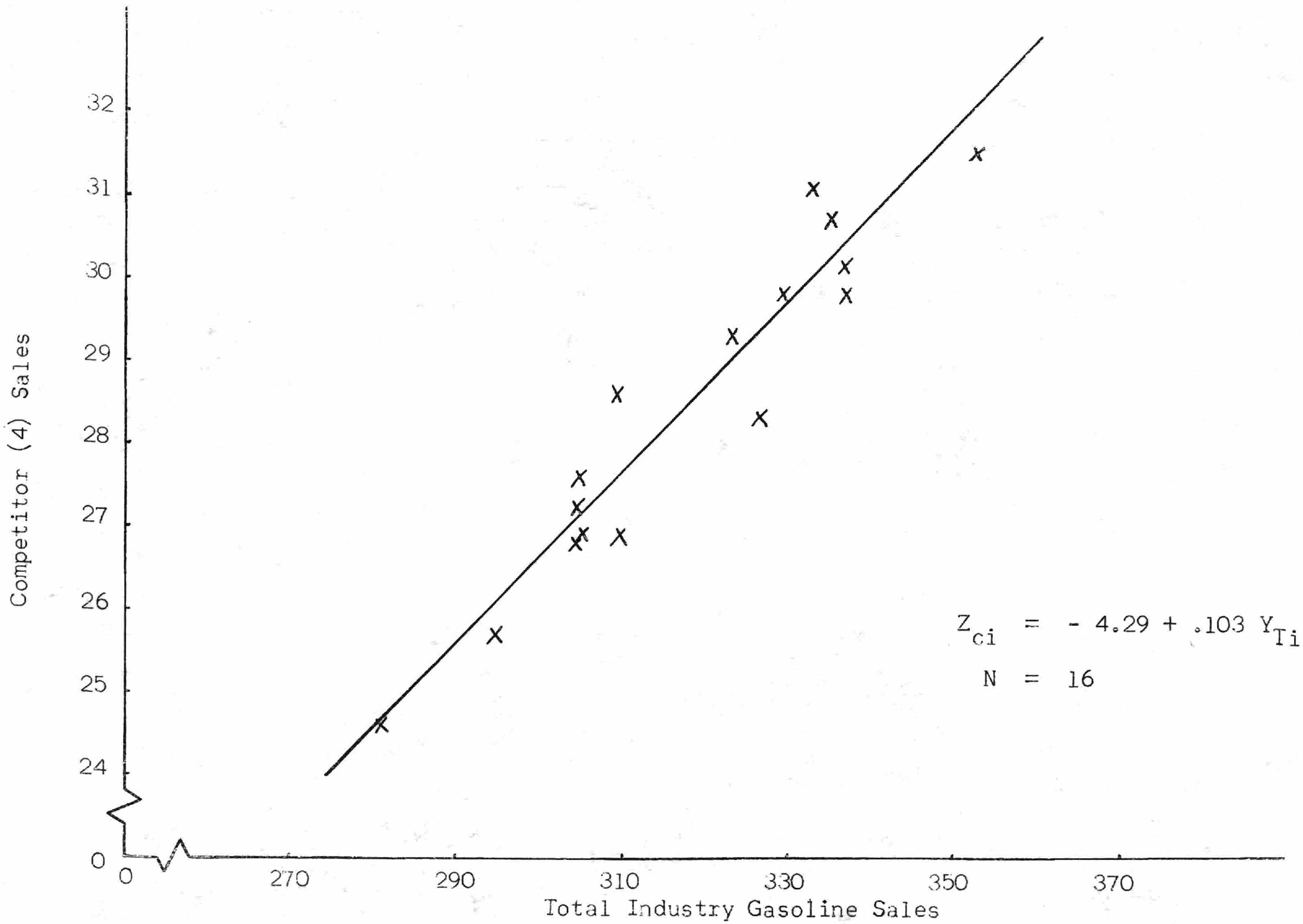

Figure 7. Competitor (4) Versus Total Industry

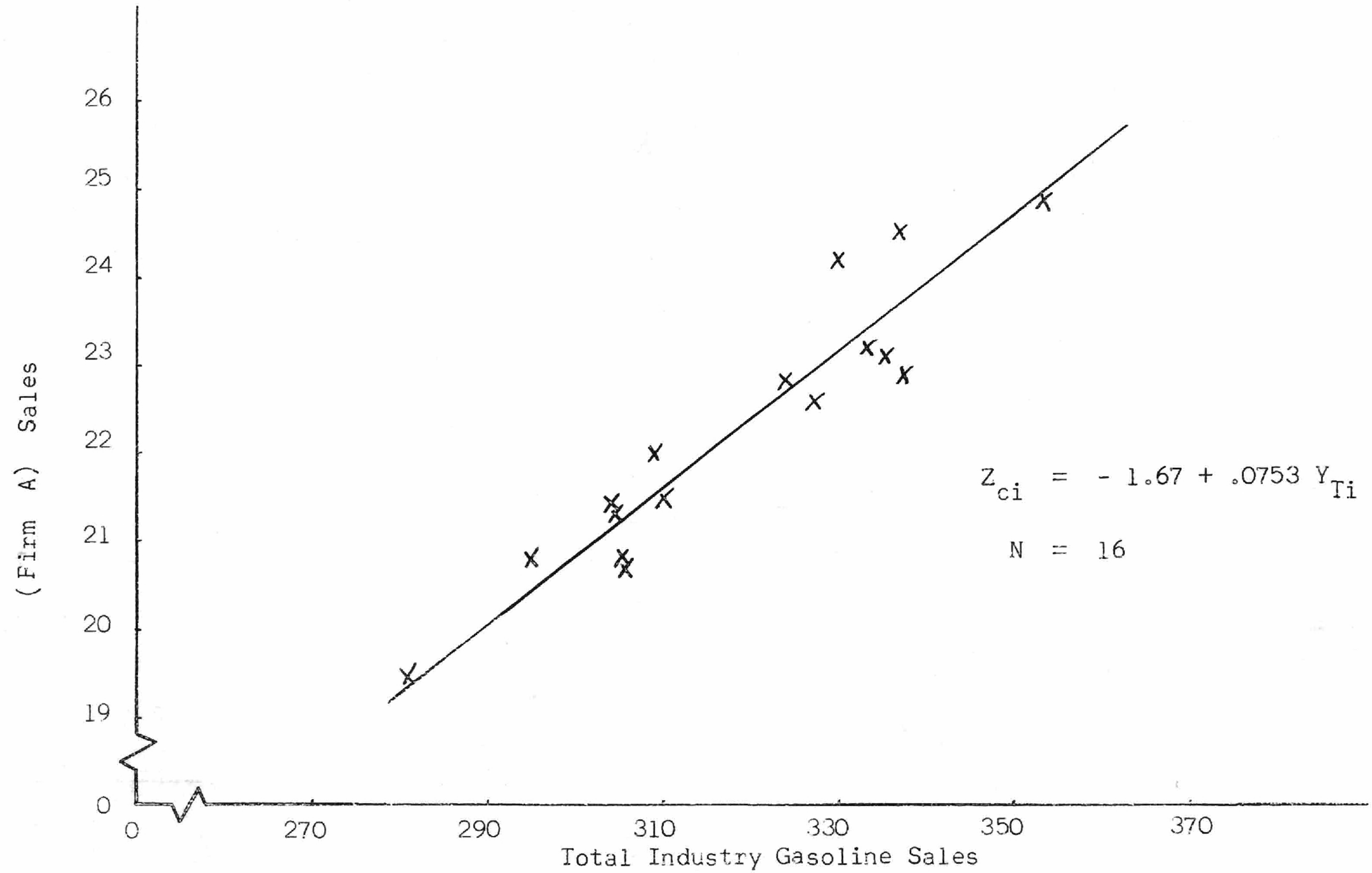

Figure 8. Firm A Versus Total Industry

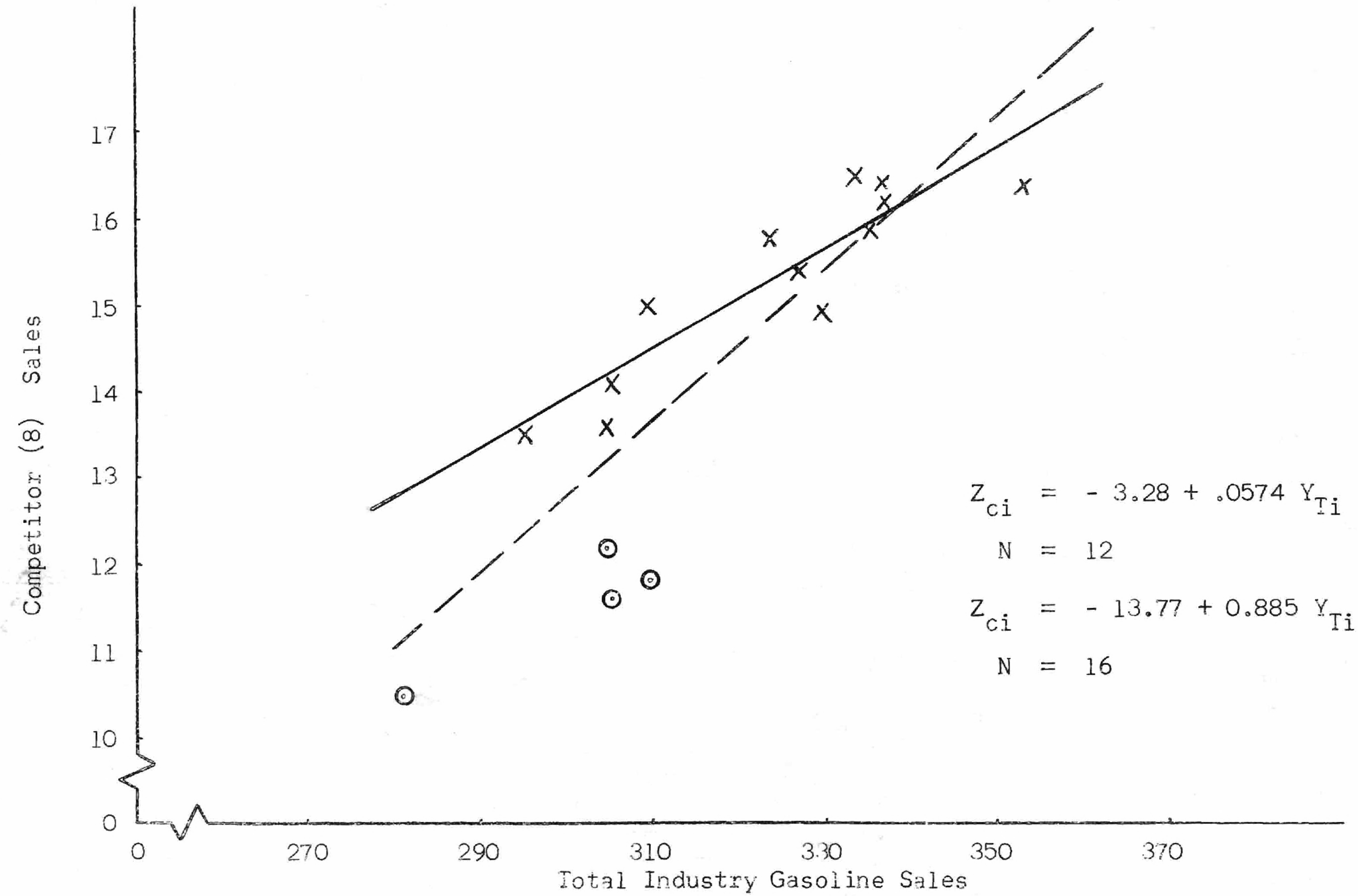

Figure 9. Competitor (8) Versus Total Industry

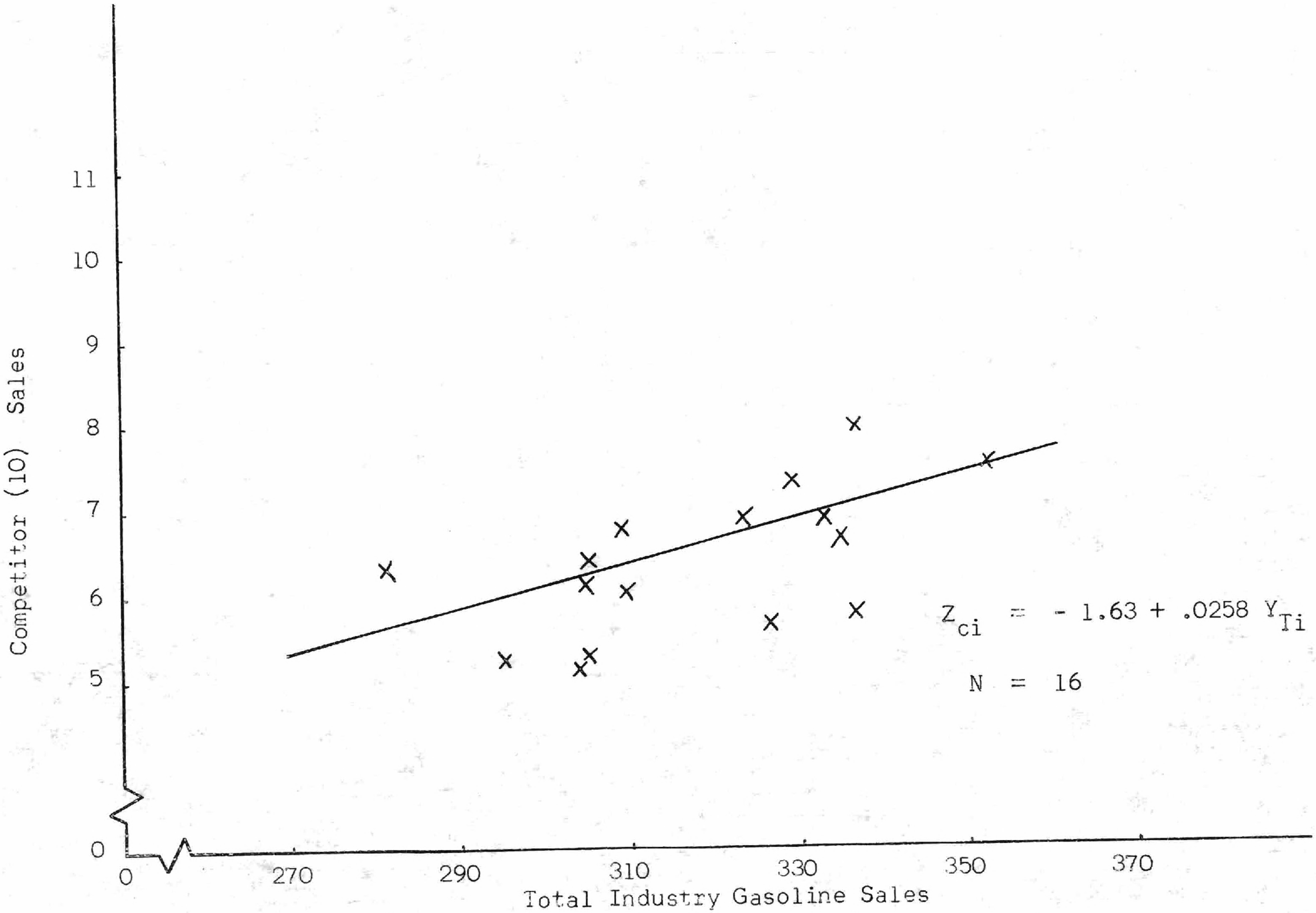

Figure 10. Competitor (10) Versus Total Industry

The proposed linear regression model in Equation (7) assumes that company sales is a linear function of total industry sales.

$$Z_{ci} = A + BY_{Ti} + \varepsilon_i \tag{7}$$

where A is the Z-intercept, B is the trend of the regression line, and ε_i is the random error independent of Y_{Ti}.

The linear regression equation for the model represented by Equation (7) is given as follows:

$$Z_{ci} = a + bY_{Ti} \tag{8}$$

where $\hat{Z}_{ci} - Z_{ci} = e_i$ = residual error for the i^{th} quarter. Using the Burroughs 220 computer routine ST02 for bivariate regression, the five following equations were developed:

Competitor (1): $$Z_{ci} = 16 + 0.107Y_{Ti} \tag{9}$$

Competitor (2): $$Z_{ci} = -4.29 + 0.103Y_{Ti} \tag{10}$$

(Firm A) : $$Z_{ci} = -1.67 + 0.0753Y_{Ti} \tag{11}$$

Competitor (8): $$Z_{ci} = -13.77 + 0.0855Y_{Ti} \tag{12}$$ [1]

$$Z_{ci} = -3.28 + 0.0574Y_{Ti} \tag{13}$$

Competitor(10): $$Z_{ci} = -1.63 + 0.0258Y_{Ti} \tag{14}$$

---

[1]Equation (12) was developed from 16 observations while Equation (13) was developed from 12 observations (See Figure 9). The reasons will be discussed in a subsequent paragraph.

The simple correlation coefficients and error statistics associated with each competitor and Firm A are listed in Table 10 and calculated in Appendix II. Since the average prediction error of 8.75 per cent, supported by a low correlation coefficient of 0.58, exceeds the previously stated criterion of five per cent, it was felt that Equation (14) was unsuitable for forecasting competitor (10)'s sales. This equation is not included in the ultimate model.

Table 10. Correlation and Error Statistics

Equation and Related Competitor	Simple Correlation Coefficient	Average Error	Average Error,%	Range of Error
(9), Competitor (1)	0.82	1132	2.26	5161
(10),Competitor (4)	0.96	490	1.72	2170
(11),(Firm A)	0.95	351	1.58	1632
(12),Competitor (8)	0.86	818	5.69	3250
(13),Competitor (8)	0.91	363	2.37	1350
(14),Competitor (10)	0.58	553	8.75	2240

The t-test from Chapter III will be used to test the degree of confidence one may have in predicting sales for the individual companies. The t-statistic with 14 degrees of freedom at the 98 per cent confidence level is 2.624; the t-statistic for Equation (13) with 10 degrees of freedom is 2.764. The t-values are listed in Table 11 and calculated in Appendix II.

Table 11. t-values for Equations (9) Through (13)

Equation and Related Competitor	t-values	Significance at $\alpha/2$
(9), Competitor (1)	5.30	Yes, at $\alpha/2 = 0.0005$
(10), Competitor (4)	11.80	Yes, at $\alpha/2 = 0.0005$
(11), (Firm A)	11.20	Yes, at $\alpha/2 = 0.0005$
(12), Competitor (8)	6.32	Yes, at $\alpha/2 = 0.0005$
(13), Competitor (8)	6.92	Yes, at $\alpha/2 = 0.0005$

A closer observation of the sales data of Figure 9 may explain the reason for developing two equations for competitor (8) instead of one. The four encircled points represent the sales for the four quarters of 1958. The subsequent sales from 1959 through 1961 are considerably higher than the 1958 sales. Further investigation revealed that competitor (8), during the last quarter of 1958, initiated a promotional campaign consisting of new methods of advertising and product upgrading. Equation (12), using the data before and after the initiation of the promotional campaign, had an average prediction error of 5.69 per cent; Equation (13), considering only the data after the promotional campaign was begun, had an average error of 2.37 per cent. Although both equations indicate a comparatively high degree of confidence as illustrated by Table 11, Equation (13) would serve as the better predictor because of the lower average error. This selection was also based on the assumption that the promotional efforts were the major cause of the sales increase. No tests were made to support this assumption.

The 95 per cent confidence intervals for three values of Y_{Ti} in the equation representing Firm A are listed in Table 12 and calculated in Appendix II. The equation used in Appendix II to determine these intervals may also be used to calculate similar intervals for the equations representing Firm A's competitors.

Table 12. 95 Per Cent Confidence Intervals

Y_{Ti}	Confidence Interval at $\alpha/2 = 0.025$
290	20.17 ± 0.244
320	22.43 ± 0.229
350	24.69 ± 0.249

Because of the low values of average percentage error, supported by the high correlation coefficients and t-values, the combined forecasting model, illustrated by Figure 11, may be used to forecast the short-run competitive position of Firm A relative to the selected competitors and the total industry sales.

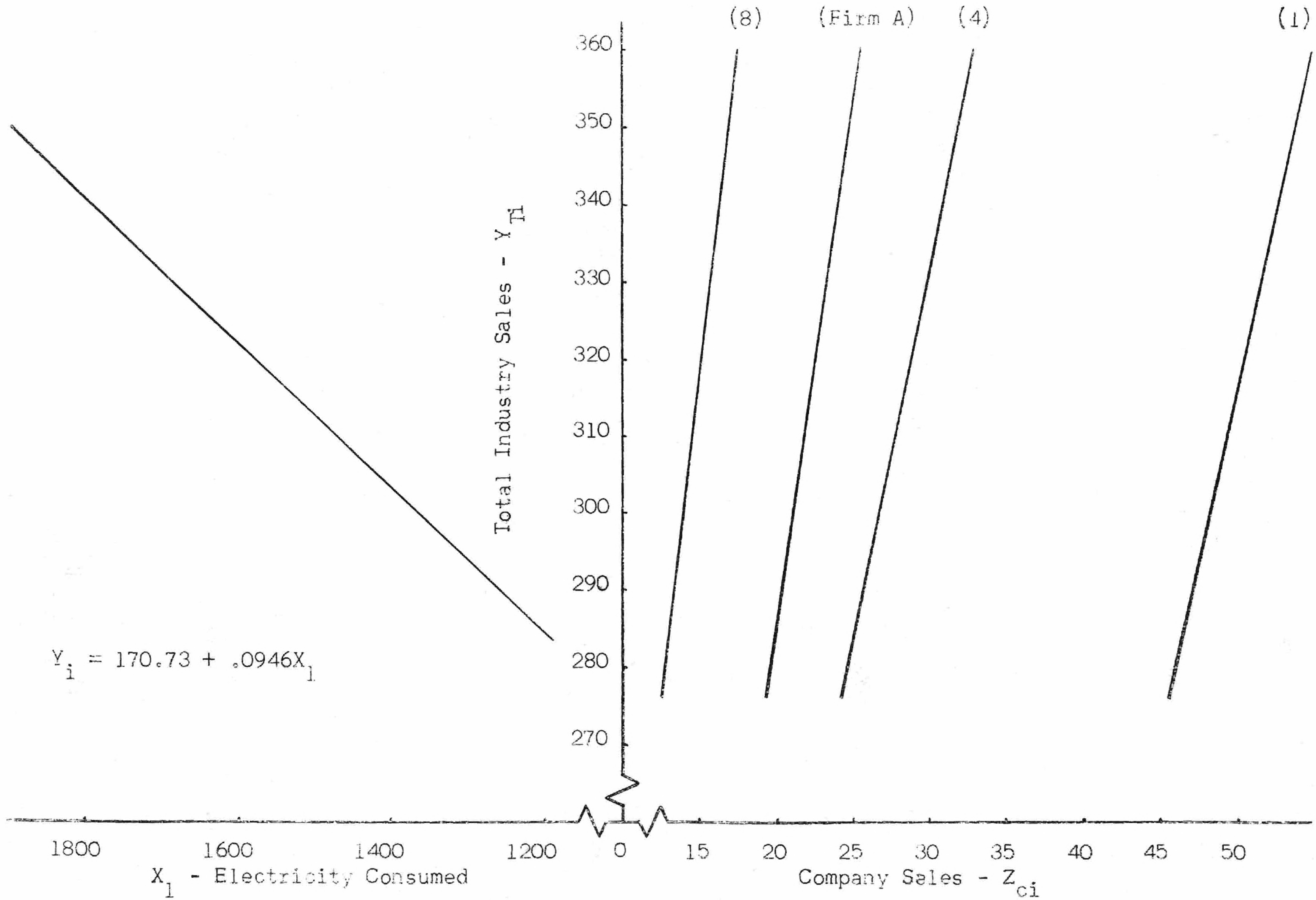

Figure 11. Combined Model

CHAPTER V

SUMMARY OF CONCLUSIONS AND DISCUSSION

Results

The results of this study were obtained using data on the total industry gasoline sales for the past five years and selected company gasoline sales for the past four years. The data were confined to the State of Georgia and analyzed quarterly.

The results of this study are as follows:

(1) Total Industry-Economy Relationship:

(a) The economic indicators telephones in service and total personal income were not found to be significant. Electricity consumed was found to be significant at the 0.0005 level.

(b) The reduced regression Equation (5) had a correlation coefficient of plus 0.89 and an average prediction error of 2.42 per cent. This equation is

$$\hat{Y}_{Ti} = 170.75 + .0946X_1 \tag{5}$$

(c) Electricity consumed was found to be a coincident indicator for the selected five-year period, necessitating the development of a technique for estimating it one quarter in advance. This was accomplished with base series and demand ratios resulting in an average prediction error of 2.96 per cent.

(2) Total Industry-Company Relationships:

(a) Competitor (10) was the only firm whose sales did not satisfactorily correlate with total industry sales.

(b) The correlation coefficients of the equations representing Firm A, Competitors (1), (4), (8), and (10) are plus 0.95, 0.82, 0.96, 0.91, and 0.58 respectively.

(c) The average per cent prediction errors for the same equations are 1.58, 2.26, 1.72, 2.37, and 8.75 respectively.

Conclusions

Equation (5) may be used to predict the total industry gasoline sales one quarter in advance. The equations that may be used in predicting the corresponding company sales are listed below, opposite their respective companies:

(Firm A): $$Z_{ci} = -1.67 + 0.0753Y_{Ti} \qquad (11)$$

Competitor (1): $$Z_{ci} = 16 + 0.107Y_{Ti} \qquad (9)$$

Competitor (4): $$Z_{ci} = -4.29 + 0.103Y_{Ti} \qquad (10)$$

Competitor (8): $$Z_{ci} = -3.28 + 0.057Y_{Ti} \qquad (13)$$

The per cent average prediction errors of these equations are within the accepted limit of five per cent and the preferred limit of three per cent.

Discussion

Assuming that electricity consumed is a reasonable representation of the overall growth of the state, the high correlations obtained between the total industry sales and selected company sales support the assumption that company sales are a function of the economic growth of the state and total industry gasoline sales. However, one should not conclude that the remaining competitors would have the same degree of correlation. Each additional company would have to be separately investigated.

Although a comparatively high correlation was obtained between total industry gasoline sales and electricity consumed, the major support of this relationship is the assumed future short-run stabilization of economic conditions and technology. This is a relatively safe assumption from a short-run approach; however, a long-run analysis could possibly result in a nonlinear relationship in lieu of a linear. Parameters which may possibly alter the relationships developed in this study from a long-range basis would include energy substitutes for gasoline or electricity, transportation substitutes (rapid transit) for automobiles, or market saturation because of a limited degree of consumption. Future research from a long-run approach might determine the effect of such parameters.

Some parameters which may possibly cause short-run abnormal variations in company sales are promotional campaigns, expansions, and mergers. Control charts using both two and three standard deviations may be used in detecting abnormal variations from the trend. The 2S limits could be the warning region while the 3S limits would be the region for possible action. The 3S limits and 2S limits include 99.7 per cent and 95 per cent respectively of the normal variations in the variable being predicted.

There are two possible explanations for the low correlation of plus 0.58 for competitor (10). One is the preciseness of the data; however, this does not seem too likely since they were obtained from the same sources as the data for the other companies. The second possibility is that competitor (10) may not feel it is necessary to follow the trend of the total industry sales. This would not necessarily mean the company is not making a profit. The firm simply may not have the distribution facilities and advertising programs used by its larger competitors. As long as the company is making an adequate profit, management may be satisfied. There was no investigation to verify either of the above explanations.

The combined model, illustrated in Figure 11, will not present explanations or reasons as to why a selected competitor's sales, for a given period, increase to an unusual magnitude. It will only signify that a change, other than the normal variations, is occurring. An example of this was illustrated in the data related to competitor (8); the continuation of the increase in sales (Figure 9, page 42) after 1958 indicated that a change in one or more parameters had occurred. Further investigation, as previously stated in Chapter IV (page 46), resulted in a reasonable explanation for the change.

The demand ratios for the fourth quarter of 1961 and the base series for the first quarter of 1962 (See Table 16, Appendix II) were used to estimate electricity consumed at 1,651 million kilowatt hours. This estimate was used to forecast the total industry gasoline sales, which in turn was used to forecast the selected company sales. The results, along with the per cent deviation from the actual sales, are listed in Table 13.

Table 13. Forecasting Results for Quarter 1, 1962

Elements of Prediction and Variation	Total Industry and Company Sales				
	Y_{Ti}	$Z_{ci}(1)$	$Z_{ci}(4)$	$Z_{ci}(A)$	$Z_{ci}(8)$
Predicted Sales	326.8	50.9	29.4	23.0	15.5
Actual Sales	345.6	49.2	30.1	23.7	15.3
Residual	- 18.8	1.7	- 0.7	- 0.7	0.2
Per Cent Error	5.44	3.46	2.33	2.95	1.31

Because of the competitive policies associated with the free enterprise system, obtaining complete information about a competitor in regards to a sudden shift in his sales would be next to impossible. However, enough information should be obtainable to justify the firm's efforts in counterbalancing his competitor's actions, or whatever caused the abnormal variation. It is recommended that at the end of each quarter, the current data, in conjunction with the past data, be used to evaluate new trends and constants prior to forecasting the competitive position of the selected firm for the future quarter.

APPENDICES

APPENDIX I

SAMPLE CALCULATIONS

The matrix of the simple correlation coefficients of Equation (3) is given below as A:

$$A = \begin{bmatrix} r_{Y_T Y_T} & r_{Y_T X_1} & r_{Y_T X_2} & r_{Y_T X_3} \\ r_{X_1 Y_T} & r_{X_1 X_1} & r_{X_1 X_2} & r_{X_1 X_3} \\ r_{X_2 Y_T} & r_{X_2 X_1} & r_{X_2 X_2} & r_{X_2 X_3} \\ r_{X_3 Y_T} & r_{X_3 X_1} & r_{X_3 X_2} & r_{X_3 X_3} \end{bmatrix}$$

Utilizing the data listed in Table 3, the A matrix becomes

$$A = \begin{bmatrix} 1.00 & .892 & .776 & .764 \\ .892 & 1.00 & .846 & .845 \\ .776 & .846 & 1.00 & .984 \\ .764 & .845 & .984 & 1.00 \end{bmatrix}$$

The inverse of Matrix A is as follows:

$$A^{-1} = \begin{bmatrix} 5.020 & -4.242 & -1.923 & 1.645 \\ -4.242 & 3.136 & .042 & -2.871 \\ -1.913 & .042 & 32.475 & -30.548 \\ 1.645 & -2.871 & -30.548 & 32.258 \end{bmatrix}$$

Now let

$$a_{ij} = (r_{ij})^{-1} = A^{-1} \quad \text{for the } i^{th} \text{ row}$$
$$\text{and the } j^{th} \text{ column.}$$

A^{-1} is the inverse of the simple correlation coefficients. The elements of a_{ij} will be used to compute the following statistics:

Multiple Regression Coefficients

$$b_k = \frac{-\,a_{ij}\,\sigma_{Y_T}}{a_{11}\,\sigma_k}, \quad \text{for} \quad k = 1, 2, 3 \quad \text{and} \quad j = 2, 3, 4,$$

and σ_k = standard deviation of the k^{th} variable, and σ_{Y_T} = standard deviation of the dependent variable Y_T.

$$b_1 = -\left[\frac{-4.242 \times 20.4}{5.020 \times 193}\right] = \frac{86.54}{968.86} = \underline{\underline{.0894}}$$

$$b_2 = -\left[\frac{-1.923 \times 20.4}{5.020 \times 1318}\right] = \frac{39.23}{6616} = \underline{\underline{.00593}}$$

$$b_3 = -\left[\frac{1.645 \times 20.4}{5.020 \times 42.8}\right] = \frac{-33.56}{214.86} = \underline{\underline{-0.154}}$$

Multiple Correlation Coefficient

$$R = \sqrt{1 - \frac{1}{a_{11}}}$$

$$= \sqrt{1 - \frac{1}{5.020}}$$

$$R = \underline{\underline{.900}}$$

Unbiased Standard Error of Regression Coefficients

$$S_{b_k} = \frac{b_k}{r_{Y_{T_k}}} \sqrt{\frac{1 - r_{ij}^2}{N - n}} \quad \text{for} \quad k = 1, 2, 3$$

and $r_{Y_{T_k}}$ = partial correlation coefficient between the dependent variable and the k^{th} independent variable with other independent variables held constant.

$$S_{b_1} = \frac{.0894}{.707} \sqrt{\frac{1 - (.707)^2}{(20 - 4)}} = .126 \sqrt{\frac{.500}{16}} = \underline{\underline{.0224}}$$

$$S_{b_2} = \frac{.00593}{.146} \sqrt{\frac{1 - (.146)^2}{(20 - 4)}} = .0406 \sqrt{\frac{.979}{16}} = \underline{\underline{.0100}}$$

$$S_{b_3} = \frac{-.156}{-.124} \sqrt{\frac{1 - (-.124)^2}{(20 - 4)}} = 1.258 \sqrt{\frac{.985}{16}} = \underline{\underline{.3107}}$$

Sample Calculations for Sums of Squares

$$\begin{aligned} \Sigma X_1^2 &= 45,912,441 \\ \underline{C} &= \underline{45,180,000} \\ SSX_1 &= \underline{\underline{732,441}} \end{aligned}$$

where C = correction factor = $n(\bar{X}_1)(\bar{X}_1)$

$$\begin{aligned} \Sigma X_1X_2 &= 546,795,460 \\ \underline{C} &= \underline{542,522,000} \\ SSX_1X_2 &= \underline{\underline{4,273,460}} \end{aligned}$$

where $C = n(\bar{X}_1)(\bar{X}_2)$

$$\Sigma X_2X_3 = 234,624,760$$
$$C = 233,541,120$$
$$SSX_2X_3 = \underline{\underline{1,083,640}}$$

where $C = n(\bar{X}_2)(\bar{X}_3)$

$$\Sigma Y^2 = 1,967,314$$
$$C = 1,959,380$$
$$SS\ Y = \underline{\underline{7,934}}$$

where $C = n(\bar{Y})(\bar{Y})$

Following the same procedure as above, the sum of squares for all four variables and their cross products are listed in Table 7.

Sums of Squares for Equation (3) - Table 4

		X_1	X_2	X_3	Y_T
SS for variables and cross products	X_1	732,441	4,273,460	139,583	69,548
	X_2	4,273,460	33,716,785	1,083,640	403,410
	X_3	139,583	1,083,640	43,119	12,993

Regression on X_1, X_2, and X_3

$$SSY_{T.123} = b_{Y1.23}\ (SS\ X_1Y) + b_{Y2.13}\ (SS\ X_2Y) + b_{Y3.12}\ (SS\ X_3Y)$$

$$= .0895(69,548) + .00593(403,410) - .154(12,993)$$

$$= 6225 + 2392 - 1949$$

$$SSY_{T.123} = \underline{\underline{6668}}$$

Total Regression

$$SS\ Y_T = \Sigma\ Y^2 - C$$

$$= 1,967,314 - 1,959,380$$

$$SS\ Y_T = \underline{\underline{7934}}$$

Regression on X_1 and X_2

$$SSY_{T.12} = b_{Y1.2}\ SS\ X_1Y_T + b_{Y2.1}\ SS\ X_2Y_T$$

where

$$b_{Y_{T.1}} = \frac{(SS\ X_1)(SS\ X_1Y_T) - SS\ X_1X_2)(SS\ X_2Y_T)}{(SS\ X_1)(SS\ X_2) - (SS\ X_1X_2)^2}$$

$$b_{Y_{T.2}} = \frac{(SS\ X_1)(SS\ X_2Y_T) - (SS\ X_1X_2)(SS\ X_1Y_T)}{(SS\ X_1)(SS\ X_2) - (SS\ X_1X_2)^2}$$

$$b_{Y_{T.1}} = \frac{(33,716,785)(69,548) - (4,273,460)(403,410)}{(732,441)(33,716,785) - (4,273,460)^2}$$

$$b_{Y_{T.1}} = \underline{\underline{.0965}}$$

$$b_{Y_{T.2}} = \frac{(732,441)(403,410) - (4,273,460)(69,548)}{(732,441)(33,716,785) - (4,273,460)^2}$$

$$b_{Y_{T.2}} = \underline{\underline{-\ .000261}}$$

$$SS\ Y_{Y.12} = .0965(69{,}548) - .000261(403{,}410)$$

$$= 6711 - 105$$

$$SS\ Y_{T.12} = \underline{\underline{6606}}$$

<u>Unbiased Error Variance of Y_{Ti} with X_1 and X_2</u>

$$S_{Y_{T.12}}^{2} = \frac{SS\ Y_T - SS\ Y_{T.12}}{n - 2}$$

$$= \frac{7934 - 6606}{18} = \frac{1328}{18}$$

$$S_{Y_{T.12}}^{2} = \underline{\underline{73.7}}$$

<u>Standard Error of Regression Coefficients</u>

$$S_{b_1}^{2} = S_{Y_{T.12}}^{2} \frac{(SS\ X_1)}{(SS\ X_1)(SS\ X_2) - (SS\ X_1X_2)^2}$$

$$S_{b_1}^{2} = 73.7 \left[\frac{732{,}441}{6{,}433{,}095{,}330{,}000} \right] = \underline{\underline{.00000839}}$$

$$S_{b_1} = \underline{\underline{.00290}}$$

$$S_{b_2}^{2} = S_{Y_{T1.2}}^{2} \frac{(SS\ X_2)}{(SS\ X_1)(SS\ X_2) - (SS\ X_1X_2)^2}$$

$$S_{b_2}^{2} = 73.7 \left[\frac{33{,}716{,}785}{6{,}433{,}095{,}330{,}000} \right] = \underline{\underline{.000386}}$$

$$S_{b_2} = \underline{\underline{.0196}}$$

t-Values for X_1 and X_2

$$t_1 = \frac{b_1}{S_{b_1}} = \frac{.0965}{.00290} = \underline{\underline{33.28}}$$

$$t_2 = \frac{b_2}{S_{b_2}} = \frac{-.0002613}{.0196} = \underline{\underline{-.0133}}$$

Simple Correlation Coefficient for Equation (5)

$$r = \frac{SS\ X_1Y_T}{\sqrt{(SS\ X_1)(SS\ Y_T)}} = \frac{69,548}{\sqrt{(732,441)(7,934)}}$$

$$r = \underline{\underline{0.894}}$$

Simple Regression Coefficients

$$b_{Y_{T.1}} = \frac{SS\ X_1Y_T}{SS\ X_1}$$

$$b_{Y_{T.1}} = \frac{69,548}{732,441} = \underline{\underline{.0946}}$$

Unbiased Standard Error of Estimate of Y_{Ti} with Equation (3)

$$S_{Y_{T.123}}^2 = \frac{SS\ Y_T - SS\ Y_{123}}{n - 2}$$

$$= \frac{7924 - 6668}{18}$$

$$S_{Y_{T.123}}^{2} = \frac{1266}{18} = \underline{\underline{70.33}}$$

$$S_{Y_{T.123}} = \sqrt{70.33} = \underline{\underline{8.39}}$$

where total SS = SS Y_T

Regression on X_1, X_2, and X_3 = SS $Y_{T.123}$

<u>Unbiased Standard Error of Estimate of Y_{Ti} with Equation (5)</u>

$$SS\ Y_T = 7{,}934$$

$$SS\ X_1Y_T = 69{,}548$$

$$SS\ X_1 = 732{,}441$$

$$SS\ Y_{T.1} = \text{Total SS} - \frac{(SS\ X_1Y_T)^2}{(SS\ X_1)}$$

$$= 7{,}934 - \frac{(69{,}548)^2}{732{,}441} = \underline{\underline{1330}}$$

$$S_{Y_{T.1}}^{2} = \frac{SS\ Y_{T.1}}{n - 2} = \frac{1330}{18} = \underline{\underline{73.9}}$$

$$S_{Y_{T.1}} = \sqrt{73.9} = \underline{\underline{8.6}}$$

<u>95 Per Cent Confidence Interval</u>

$$CI = \hat{Y}_i \pm t_{\alpha/2;N-2}\, S_{Y.1} \sqrt{1 + \frac{1}{N} + \frac{(X_i - \bar{X}_1)^2}{(SS\ X_1)}}$$

$$= \hat{Y}_i \pm (2.101)(8.6) \sqrt{1 + \frac{1}{20} + \frac{(X_i - 1503)^2}{732,441}}$$

$$CI = \hat{Y}_i \pm 18.07 \sqrt{1.05 + \frac{(X_i - 1503)^2}{732,441}}$$

where $t_{.05/2;18} = 2.101$

X_1	$\hat{Y}_i$	CI at $\alpha = .05$	
1300	294	275,	313
1600	322	303.4,	340.6
1900	350	329.7,	370.3

Table 14. Calculation of Residual for Equation (3)

Quarter	Predicted $\hat{Y}_i$	Actual Y_i	Residual Error
1957 - 1	287.0	278.7	+ 8.3
2	293.6	302.0	- 8.4
3	305.3	301.1	+ 4.2
4	285.9	288.3	- 2.4
1958 - 1	291.4	281.0	+ 10.4
2	292.1	305.3	- 13.2
3	312.2	309.9	+ 2.3
4	302.9	304.6	- 1.7
1959 - 1	304.7	295.1	+ 9.6
2	311.8	326.8	- 15.0
3	333.1	336.9	- 3.8
4	313.6	304.2	+ 9.4
1960 - 1	313.6	304.9	+ 8.7
2	320.9	335.1	- 14.2
3	345.5	332.7	+ 12.8
4	322.4	323.6	- 1.2
1961 - 1	318.3	309.5	+ 8.8
2	325.2	336.7	- 11.5
3	347.3	352.9	- 5.6
4	335.2	329.8	+ 5.4

Table 15. Calculation of Residual for Equation (5)

Quarter	Predicted $\hat{Y}_i$	Actual Y_i	Residual Error
1957 - 1	286.3	279.0	+ 7.3
2	294.3	302.0	- 7.7
3	307.6	301.1	+ 6.5
4	287.7	288.3	- 0.6
1958 - 1	292.0	281.0	+ 11.0
2	292.7	305.3	- 12.3
3	312.1	309.9	+ 2.2
4	301.9	304.6	- 2.7
1959 - 1	303.3	295.1	+ 8.2
2	309.7	326.8	- 17.1
3	332.7	336.9	- 4.2
4	312.3	304.2	+ 8.1
1960 - 1	312.6	304.9	+ 7.7
2	319.2	335.1	- 15.9
3	346.4	332.7	+ 13.7
4	323.7	323.6	- 0.1
1961 - 1	318.8	309.5	+ 9.3
2	326.2	336.7	- 10.5
3	348.6	352.9	- 4.3
4	332.1	329.8	+ 2.3

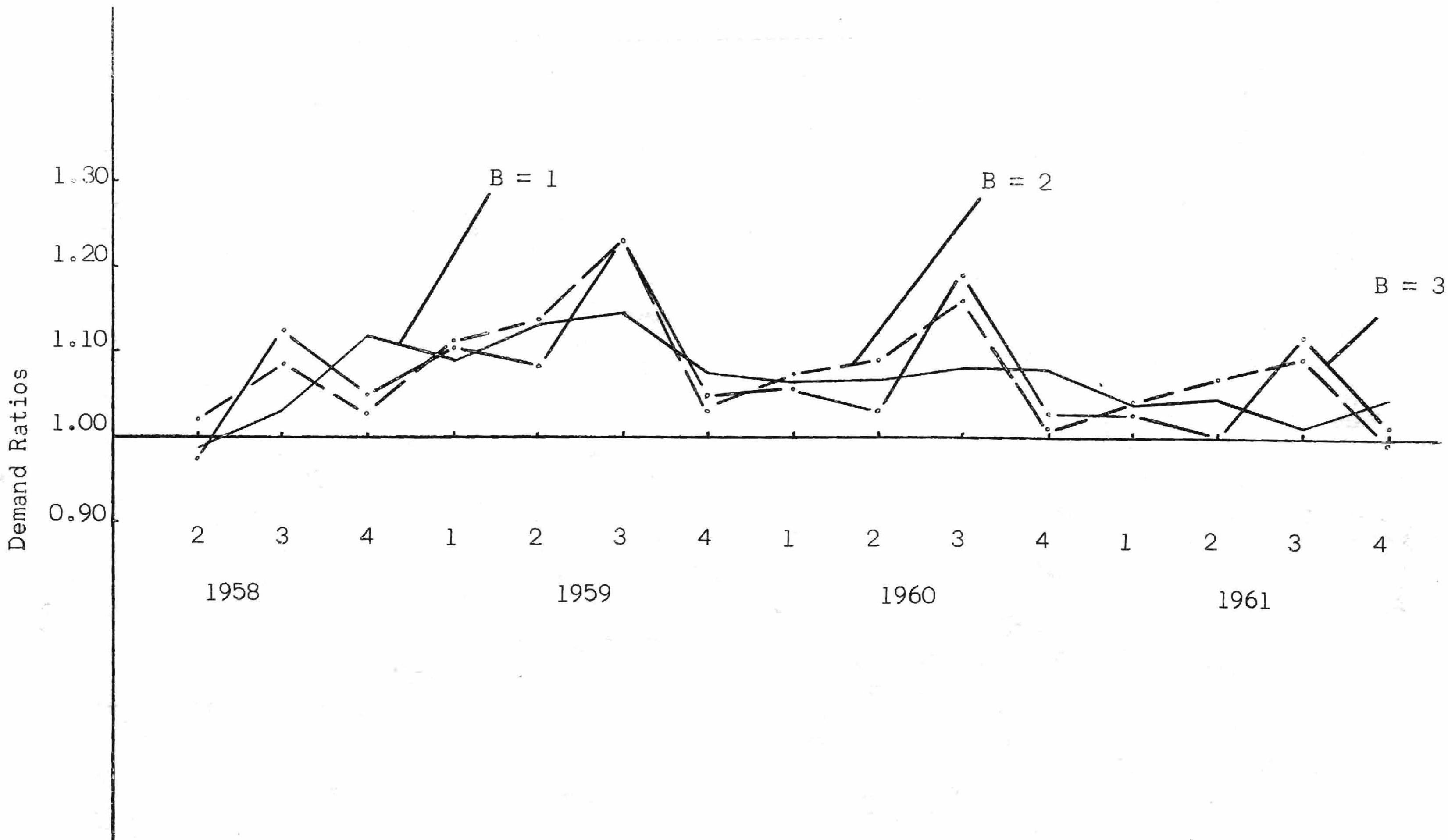

Figure 12. Demand Ratios for Selected Base Series

Table 16. Projection of Electricity Consumed

Quarter	Current Demand	Past Demand for Corresponding Quarter, B = 1	Demand Ratio	Forecast	Error
1958 - 1	1,281	1,221	1.0491		
2	1,289	1,306	0.9870	1,370	+ 81
3	1,494	1,446	1.0331	1,427	- 67
4	1,386	1,236	1.1213	1,277	- 109
1959 - 1	1,401	1,281	1.0936	1,436	+ 35
2	1,468	1,289	1.1388	1,410	- 58
3	1,712	1,494	1.1459	1,701	- 11
4	1,496	1,386	1.0793	1,588	+ 92
1960 - 1	1,499	1,401	1.0699	1,512	+ 13
2	1,569	1,468	1.0688	1,571	+ 2
3	1,856	1,712	1.0841	1,830	- 26
4	1,616	1,496	1.0802	1,622	+ 6
1961 - 1	1,565	1,499	1.0440	1,619	+ 54
2	1,643	1,569	1.0471	1,638	- 5
3	1,880	1,856	1.0129	1,943	+ 63
4	1,705	1,616	1.0550	1,637	- 68
1962 - 1		1,565		1,651	

APPENDIX II

SAMPLE CALCULATIONS - CHAPTER IV

Table 17. Calculation of Residual for Equation (9)

Quarter	Predicted $\hat{Z}_{ci}$	Actual Z_{ci}	Residual Error
1958 - 1	45,799	46,750	- 951
2	48,494	50,740	- 2246
3	49,006	50,051	- 1044
4	48,415	48,559	+ 145
1959 - 1	47,364	46,923	+ 444
2	50,890	52,289	- 1400
3	52,003	51,817	+ 183
4	48,375	46,462	+ 1915
1960 - 1	48,457	47,633	+ 827
2	51,801	53,454	- 1649
3	51,535	51,424	+ 115
4	50,532	49,269	+ 1360
1961 - 1	48,959	47,891	+ 1069
2	51,979	51,536	+ 439
3	53,785	54,283	- 495
4	51,217	49,477	+ 1737

Table 18. Calculation for Residual for Equation (10)

Quarter	Predicted $\hat{Z}_{ci}$	Actual Z_{ci}	Residual Error
1958 - 1	24,800	24,594	+ 206
2	27,095	26,933	+ 162
3	27,554	26,855	+ 699
4	27,025	26,779	+ 246
1959 - 1	26,107	25,697	+ 410
2	29,312	28,300	+ 1012
3	30,406	29,825	+ 582
4	26,989	27,211	- 222
1960 - 1	27,063	27,564	- 501
2	30,201	30,656	- 455
3	29,941	31,063	- 1122
4	28,969	29,315	- 346
1961 - 1	27,514	28,584	- 1070
2	30,380	30,702	- 322
3	32,237	31,469	+ 768
4	29,628	29,782	- 154

Table 19. Calculation of Residual for Equation (11)

Quarter	Predicted $\hat{Z}_{ci}$	Actual Z_{ci}	Residual Error
1958 - 1	19,575	19,479	+ 96
2	21,275	20,751	+ 524
3	21,610	21,529	+ 81
4	21,220	21,297	- 77
1959 - 1	20,545	20,779	- 234
2	22,905	22,625	+ 280
3	23,705	22,952	+ 753
4	21,195	21,431	- 236
1960 - 1	21,250	20,875	+ 375
2	23,559	23,095	+ 464
3	23,365	23,190	+ 175
4	22,655	22,787	- 134
1961 - 1	21,582	21,970	- 388
2	23,690	24,477	- 787
3	25,045	24,926	+ 119
4	23,139	24,230	-1091

Table 20. Calculation of Residual for Equation (12)

Quarter	Predicted $\hat{Z}_{ci}$	Actual Z_{ci}	Residual Error
1958 - 1	11,230	10,526	+ 704
2	13,119	11,565	+ 1554
3	13,512	11,794	+ 1718
4	11,306	12,247	- 941
1959 - 1	12,292	13,528	- 1236
2	15,065	15,427	- 362
3	16,064	16,200	- 136
4	13,030	13,677	- 647
1960 - 1	13,091	14,051	- 960
2	15,877	15,920	- 43
3	15,635	16,492	- 857
4	14,756	15,806	- 1050
1961 - 1	13,479	15,006	- 1527
2	16,042	16,371	- 329
3	17,805	16,365	+ 1440
4	15,351	14,984	+ 367

Table 21. Calculation of Residual for Equation (13)

Quarter	Predicted $\hat{Z}_{ci}$	Actual Z_{ci}	Residual Error
1958 - 1			
2			
3			
4			
1959 - 1	13,690	13,528	+ 162
2	15,449	15,427	+ 22
3	16,049	16,200	- 151
4	14,173	13,677	+ 496
1960 - 1	14,214	14,051	+ 163
2	15,939	15,920	+ 19
3	15,795	16,492	- 697
4	15,261	15,806	- 545
1961 - 1	14,461	15,006	+ 545
2	16,037	16,371	- 334
3	17,062	16,365	+ 697
4	15,622	14,984	+ 638

Table 22. Calculation of Residual for Equation (14)

Quarter	Predicted $\hat{Z}_{ci}$	Actual Z_{ci}	Residual Error
1958 - 1	5474	6320	- 846
2	6020	6437	- 417
3	6129	6045	+ 84
4	6003	6143	- 140
1959 - 1	5785	5263	+ 522
2	6551	5617	+ 934
3	6812	5762	+ 1050
4	5992	5179	+ 813
1960 - 1	6011	5289	+ 722
2	6765	6664	+ 101
3	6701	6923	- 222
4	6469	6885	- 416
1961 - 1	6119	6778	- 659
2	6795	7999	- 1204
3	7256	7523	- 267
4	6626	7266	- 640

Table 23. Components for Statistical Evaluation of Equations (9) Through (14)

Entries	Equation (9)	(10)	(11)	(12)	(13)	(14)
$\Sigma\ Z_{ci}$	798.55	455.32	356.4	229.97	183.84	102.09
$\bar{Z}_{ci}$	49.91	28.46	22.28	14.37	15.32	6.32
$\Sigma\ Z_{ci}^2$	39,944	13,020	7973.3	3362.7	2829.1	661.96
$(\Sigma\ Z_{ci})^2/n$	39,855	12,957	7939.0	3305.4	2816.4	651.39
$\Sigma\ Y_{Ti}$	5088.74	5088.74	5088.74	5088.74	3888.1	5088.74
$\bar{Y}_{Ti}$	318.0	318.0	318.0	318.0	324.0	318.0
$\Sigma\ Y_{Ti}^2$	1,623,873	1,623,873	1,623,873	1,623,873	1,262,969	1,623,873
$(\Sigma\ Y_T)^2/n$	1,618,454	1,618,454	1,618,454	1,618,454	1,259,777	1,618,454
$\Sigma\ Y_{Ti}\ Z_{ci}$	254,576	145,370	113,760	73,621	59,749	32,609
$(\Sigma\ Y_T)(\Sigma\ Z_{ci})/n$	253,976	144,813	113,351	73,141	59,566	32,469
$SS\ Y_{Ti}$	5419	5419	5419	5419	3192	5419
$SS\ Z_{ci}$	89	63	34.3	57.3	12.7	10.57
$SS\ Y_{Ti}\ Z_{ci}$	600	557	409	480	183.0	140

The elements listed in Table 23 will be used to determine the following statistics:

Unbiased Standard Error of Estimate

$$\text{Standard error for Equation (9)} = {}^{*}S_{Z_{ci(9)}} = \sqrt{\frac{SS\,Z_{ci} - \dfrac{(SSY_{Ti}Z_{ci})^2}{SS\,Y_{Ti}}}{n-2}}$$

Regression Coefficients

$$\text{The regression coefficient for Equation (9)} = b_{(9)} = \frac{SS\,Y_{Ti}\,Z_{ci}}{SS\,Y_{Ti}}$$

Standard Error of Regression Coefficients

The standard error of the regression coefficient for Equation (9) =

$$S_{b(9)} = \sqrt{\frac{S^2_{Z_{ci(9)}}}{SS\,Y_{Ti}}}$$

t-Values for Table 12

$$\text{The t-values for Equation (9)} = t_{(9)} = \frac{b(9)}{S_{b(9)}}$$

Table 24. Statistics for Equations (9) Through (14)

Statistic	Equations					
	(9)	(10)	(11)	(12)	(13)	(14)
$S_{Z_{ci}}$	1.608	0.638	0.495	1.028	0.470	0.705
b	0.111	0.103	0.0753	0.0885	0.0574	0.0258
S_b	0.0219	0.0087	0.00675	0.0140	0.0083	0.0096
t	6.42	11.80	11.20	6.32	6.92	------

*The numbers enclosed in the parentheses of the related subscript represent the equation being evaluated.

95 Per Cent Confidence Interval

$$C. I. = \hat{Z}_{ci} \pm t_{\alpha/2;n-2} \, S_{Z_{ci(11)}} \sqrt{1 + \frac{1}{n} + \frac{(Y_{Ti} - \overline{Y}_{Ti})^2}{SS \, Y_{Ti}}}$$

where $t_{\alpha/2;14} = 2.145$

Y_{Ti}	$\hat{Z}_{ci}$	Confidence Interval
290	20.17	19.004, 21.336
320	22.43	21.337, 23.523
350	24.69	23.504, 25.876

Total Standard Error of Equation (11) - Firm A

$$\hat{Y}_{Ti} = 170.73 + 0.0946X_1 \tag{5}$$

$$Z_{ci} = -1.67 + .0753Y_{Ti} \tag{11}$$

by substitution

$$Z_{ci} = -1.67 + .0753(170.73 + .0946X_1)$$

$$Z_{ci} = 11.19 + .00712X_1$$

Since the variance of a constant is zero, $S_{11.19} = 0$; therefore

$$S_{Z_{ci}.\hat{Y}_{Ti}}^2 = (.00712)^2 \, S_{X_1}^2$$

$$S_{Z_{ci}.\hat{Y}_{Ti}} = \underline{\underline{1.372}}$$

BIBLIOGRAPHY

1. American Management Association, "Sales Forecasting: Uses, Techniques, and Trends," Special Report 16, New York, 1956, pp. 141-159.

2. American Petroleum Institute, Petroleum Facts and Figures, New York, 1961, p. 156.

3. Ibid., p. 141.

4. Brown, Robert G., Statistical Forecasting for Inventory Control, McGraw-Hill Book Company, Inc., New York, 1959.

5. Bass, F. M., Buzzell, R. D., Greene, M. R., Pessemier, E. A., Layer, W., Shawver, D. L., Shuchman, A., Theodore, C. A., and Wilson, G. W., Mathematical Models and Methods in Marketing, Richard D. Irwin, Inc., Homewood, Illinois, 1961, pp. 482-514.

6. Magee, John F., Production Planning and Inventory Control, McGraw-Hill Book Company, Inc., New York, 1958, pp. 105-126.

7. Moore, G. H., Business Cycle Indicators, Volume 1, Princeton University Press, 1961, p. 181.

8. Goldberg, Melvin, "How Top Management Forecasts the Future," Dun's Review and Modern Industry, September, 1958, pp. 33-35, p. 129.

9. American Management Association, "Materials and Methods of Sales Forecasting," Special Report 27, New York, 1957, pp. 140-153.

10. Dakin, E. F., and Dewey, E. R., Cycles, The Science of Prediction, Henry Holt and Company, New York, 1947, pp. 140-153.

11. Gordon, Robert A., Business Fluctuations, Second Edition, Harper and Brothers, New York, 1961.

12. Lindsay, Franklin A., New Techniques for Management Decision Making, McGraw-Hill Book Company, Inc., New York, 1958, pp. 12-14.

13. Klein, L. R., A Textbook of Econometrics, Row, Peterson and Company, Evanston, Illinois, 1953, pp. 249-276.

14. Clark, Colin, "Wider, Quicker Business Forecasts Brewed by Econometric Institute," Business Week, August, 1959, p. 28.

15. Stillson, Paul, and Arnoff, L. E., "Product Search and Evaluation," Journal of Marketing, Volume 22, No. 1, July 1957, pp. 33-39.

16. Vidale, M. L., and Wolfe, H. B., "An Operations Research Study of Sales Response to Advertising," Operations Research, Volume 5, No. 3, June, 1957, pp. 370-381.

17. Lindsay, op. cit., pp. 85-95.

18. Forrester, J. W., "Industrial Dynamics," Harvard Business Review, July, 1958, pp. 37-66.

19. Roos, Charles F., "Survey of Economic Forecasting Techniques," Econometrics, Volume 25, No. 4, October, 1955, pp. 369-379.

20. Moore, G. H., op. cit., p. 162.

21. Moore, G. H., op. cit., pp. 45-110.

22. Sommers, A. T., "Forecasting and the Diffusion Indexes," Analysts Journal, Volume 14, February, 1958, pp. 23-27.

23. Platt, H. M., "Economic Indicators--How to Use Them in Business Forecasting," The Management Review, April, 1959, pp. 9-13, pp. 67-73.

24. Reinfeld, Nyles V., Production Control, Pentice-Hall, Inc., Englewood Cliffs, N. J., 1959, pp. 98-101.

25. Roos, op. cit., p. 383.

26. Ibid., p. 388.

27. Ferber, Robert, "On the Reliability of Responses Secured in Sample Surveys," Journal of American Statistical Association, Volume 50, 1955, pp. 788-810.

28. Klein, L. R., An Introduction to Econometrics, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1962, pp. 236-244.

29. Davis, Edward W., Forecasting the Demand for Hospital Supply Items, M. S. Thesis, Georgia Institute of Technology, January, 1960, pp. 15-27.

30. Cotter, J. E., "Sales Forecasting," Advanced Management, September, 1959, pp. 16-22.

31. Sonnendecker, J. P., A Model for Forecasting the Whole Blood Requirements of a Hospital Blood Laboratory, M. S. Thesis, Ohio State University, 1958.

32. Eilon, Samuel, Elements of Production Planning and Control, The Macmillan Company, New York, 1962, pp. 99-139.

33. Weinberg, R. S., "Multiple Factor Break-Even Analysis: The Application of Operations Research Techniques to a Basic Problem of Management Planning and Control," Operations Research. Volume 4, No. 2, April, 1956, pp. 152-186.

34. Snedecor, George W., Statistical Methods, The Iowa State College Press, Ames, Iowa, 1959, pp. 413-446.

35. Brown, op. cit., pp. 128-135.